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Effect of stiffener on nonlinear characteristic behavior of a rectangular plate: A single mode approach

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ABSTRACT

In this paper, nonlinear dynamic of an excited square plate with a single stiffener is investigated using a single mode approach. In a first part, we present an exact solution of the buckling mode of the stiffened plate under uni-axial compression. The differential equation of the deflection surface of the rectangular plate with different boundary conditions is used and the effect of the stiffener upon natural frequency of the plate is examined. In a second part, we analyze the dynamic of the excited mode near the principal resonance. Specifically, we study the influence of stiffener on nonlinear characteristic behavior of the excited plate.

1. Introduction

Stiffened plates are widely used in structures in order to increase their strength and enhance their performance. Applications include aircraft, helicopters and spacecraft for quoting a few. The problem of stiffened plates has been studied following three different trends. One approach consists of replacing the stiffened plate by an 'equivalent' orthotropic plate after the stiffeners are smeared out, in an energetic sense, over the entire surface of the plate (Brush and Almroth, 1975; Mcfarland et al., 1972). The second approach is based on energy consideration and treats the contribution of the plate and the stiffener separately (Bulson, 1969; Liew and Wang, 1990). The third approach is the analytical method for equally spaced stiffeners by the analytical finite difference calculus (Wah and Calcote, 1970; Bleich, 1952). Elishakoff et al. (1995) investigated the effect of small structural irregularity, due to the misplacement of stiffeners or interior supports, on both the buckling load and the buckling mode of the rib-stiffened plate. Tao et al. (2004) analyzed the nonlinear dynamic buckling of stiffened plates under the in-plane impact load, whereas Chen et al. (2006) studied the effect of nonlinear contact upon natural frequency of delaminated stiffened composite plate.

In the present work, we study analytically nonlinear dynamic of a simply supported stiffened plate in a single mode approach. A similar approach was used to analyze nonlinear flexural vibration of a thin circular ring (Rougui et al., 2008). Specifically, we examine the influence of a single-rib stiffener on the natural frequency of the buckling mode and on the nonlinear behavior of the stiffened plate under uni-axial compression.

In the first part of the paper, the differential equation of the deflection surface of the rectangular plate (Chia, 1980) with different boundary conditions is used. Here, we focus our attention on the case of a simple support under the rib (Mcfarland et al., 1972). The exact solution to the problem is obtained and an ordinary parametric nonlinear differential equation of the buckling mode shape is established using a Galerkin's method. Therefore, the effect of the stiffener upon natural frequency of the stiffened plate is examined. In the second part, we study the nonlinear behavior of the buckling mode shape considering the Mathieu–Duffing oscillator model obtained via the Galerkin's method. Using the multiple scale technique (Nayfeh and

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Mook, 1979), the slow flow system describing the modulation of the amplitude and the phase of the periodic response of the considered mode is derived. The influence of the location of the stiffener on the hardening and softening behavior are examined in the parameter plane corresponding to nonlinear component versus torsional rigidity.

2. Formulation

2.1. Linear case

We consider a rib-stiffened plate subjected in its mid-plane to uniform compression P in the x-direction; see Fig. 1. From von Karman-type equations for the thin plate, we obtain the following equation of the deflection surface of the plate

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+P\frac{\partial^2 w}{\partial x^2}=0 \tag{1}$$

where w is the transverse displacement, downward positive and D is the flexural rigidity of the plate. The in-plane excitation of the thin plate may be expressed in the form $P=P_0-P_D\cos(\Omega t)$. We introduce the transformations $\bar{x}=\frac{x}{a},\bar{y}=\frac{y}{b},\bar{w}=\frac{w}{h},\overline{P}=\frac{a^2}{D}P$ and $\lambda=\frac{a}{b}$, and for simplicity, we drop overbars. Hence, Eq. (1) can be written in the nondimensional form as

$$\frac{\partial^4 w}{\partial x^4}+2\lambda^2\frac{\partial^4 w}{\partial x^2\partial y^2}+\lambda^4\frac{\partial^4 w}{\partial y^4}+P\frac{\partial^2 w}{\partial x^2}=0 \tag{2}$$

The solution of Eq. (2) can be represented by

$$w(x,y)=X(x)\sin(\pi y) \tag{3}$$

Substituting Eq. (3) into Eq. (2) leads to

$$\frac{d^4}{dx^4}X(x)+\left(P-2\lambda^2\pi^2\right)\frac{d^2}{dx^2}X(x)+\lambda^4\pi^4X(x)=0 \tag{4}$$

The corresponding characteristic equation reads

$$s^4+\left(P-2\lambda^2\pi^2\right)s^2+\lambda^4\pi^4=0 \tag{5}$$

and the solutions are written as

$$s^2=-\left(\frac{P}{2}-\lambda^2\pi^2\right)\pm\sqrt{\frac{P}{2}\left(\frac{P}{2}-2\lambda^2\pi^2\right)} \tag{6}$$

where the roots are given by $s_1=i\beta_1, s_2=-i\beta_1, s_3=i\beta_2$ and $s_4=i\beta_2$ with $\beta_1=\sqrt{\left(\frac{P}{2}-\lambda^2\pi^2\right)+\sqrt{\frac{P}{2}\left(\frac{P}{2}-2\lambda^2\pi^2\right)}}$ and $\beta_2=\sqrt{\left(\frac{P}{2}-\lambda^2\pi^2\right)-\sqrt{\frac{P}{2}\left(\frac{P}{2}-2\lambda^2\pi^2\right)}}$.

A solution of Eq. (2) is given by

$$w(x,y)=\left(A\cos(\beta_1 x)+B\sin(\beta_1 x)+C\cos(\beta_2 x)+D\sin(\beta_2 x)\right)\sin(\pi y) \tag{7}$$

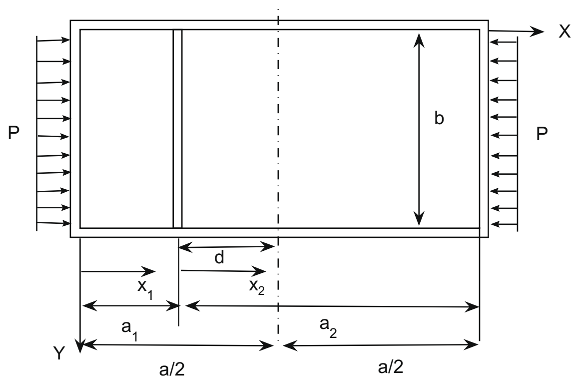

Fig. 1. Uni-axial compressed rectangular stiffened plate with a single misplaced rib.