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Body wave propagation in rotating thermoelastic media

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ABSTRACT

The present paper deals with the propagation of body waves in a homogenous isotropic, rotating, generalized thermoelastic solid. The complex cubic secular equation has been solved by using Cardano's and perturbation methods to obtain phase velocities, attenuations and specific loss factors of three attenuating and dispersive waves, which are possible to exist in such media. These wave characteristics have also been computed numerically for magnesium crystal and are presented graphically. Statistical analysis has been performed to compare the computer simulated results obtained by using both methods. This work may find applications in geophysics and gyroscopic sensors.

1. Introduction

The propagation of elastic waves in 3-D unbounded media is a classical problem. The vibrational analysis of elastic rotating structures such as beams; disks and membranes have been thoroughly addressed in literature by the authors (Advani, 1966; Advani and Bulkeley, 1969; Hashemi and Richard, 2001; Huang and Wang, 2001; Luo and Mote, 2000). These studies are generally focused to determine the natural frequencies of some particular structures under rotation. The case of small angular rotation ($\Omega \ll \omega$) has been addressed by the geophysicists to investigate surface waves and natural frequencies of the Earth (Aki and Richards, 1980). Auriault (2004) studied the body wave propagation in an infinite homogenous isotropic elastic medium rotating with uniform angular velocity with respect to a Galilean axis. The classical theory of heat conduction predicts an infinite speed of heat transportation which contradicts the physical facts. During the last three decades, non- classical theories have been developed to alleviate this paradox. Lord and Shulman (1967) incorporated a flux-rate term in Fourier's law of heat conduction in order to formulate a generalized theory that admits finite speed for thermal signals. Green and Lindsay (1972) included a temperature- rate term among the constitutive variables to develop a temperature- rate- dependent thermoelasticity that does not violate the classical Fourier's law of heat conduction in case of centrally symmetric bodies. This theory also predicts a finite speed of heat propagation. According to these non-classical theories, heat propagation has been viewed as a wave phenomenon rather than diffusion one. Chandrasekharaiah (1986) referred a wave-like thermal disturbance as 'second sound'. The actual occurrence of 'second sound' at low temperatures and small intervals of time has also been supported with experimental exhibition by the researchers (Ackerman and Overtone, 1969; Ackerman et al., 1966) in solid helium. Guyer and Krumhansal (1966) also presented a theoretical study of 'second sound' in solids.

The present paper deals with the problem of wave propagation in a homogenous isotropic, thermoelastic medium which is rotating with uniform angular velocity ($\vec{\Omega}$) about a fixed axis $\vec{e}_3$. The waves propagating in the plane ($\vec{e}_1, \vec{e}_2$) perpendicular to $\vec{e}_3$ may be affected by the Coriolis force under such a situation. In general, the wave propagation in the considered medium is found to be governed by a complex cubic polynomial secular equation which provides us three complex roots. The secular equation has

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been solved by using Cardano's and perturbation methods to obtain the roots which can be associated with three dispersive and attenuating waves namely, quasi-longitudinal (QL), quasi-transverse (QT) and thermal (T-mode) waves. In order to illustrate the analytical developments, the numerical solution of secular equation has also been carried out for magnesium like material.

2. Wave equations in rotating media

We consider a homogenous isotropic, thermoelastic solid initially at uniform temperature T_0 in the undisturbed state. The medium is assumed to be rotating with uniform angular velocity $\vec{\Omega}$ with respect to an inertial frame. The basic governing dynamical equations of linear generalized thermoelastic interactions (Hetnarski and Ignaczak, 2000) after including the Coriolis and centripetal forces, in absence of body forces and heat sources, are given by

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla.\vec{u} - \beta\nabla T = \rho(\ddot{\vec{u}} + \vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2(\vec{\Omega}\times\dot{\vec{u}})), \tag{1}$$

$$K\nabla^2 T - \rho C_e(\dot{T} + t_0\ddot{T}) = \beta T_0\nabla.(\dot{\vec{u}} + t_0\ddot{\vec{u}}), \tag{2}$$

where t_0 is the thermal relaxation time, $\vec{u}(x_1,x_2,x_3,t) = (u_1,u_2,u_3)$ is the displacement vector; $T(x_1,x_2,x_3,t)$ is the temperature change; λ,μ are Lamé's parameters; K is thermal conductivity; ρ and C_e are respectively, the density and specific heat at constant strain; $\beta = (3\lambda + 2\mu)\alpha_t$,$\alpha_t$ is the linear thermal expansion. Here superposed dot represents time differentiation.

We define the non-dimensional quantities

$$x_i' = \frac{\omega^* x_i}{c_1},\ t' = \omega^* t,\ u_i' = \frac{\rho\omega^* c_1 u_i}{\beta T_0},\ T' = \frac{T}{T_0},\quad t_0' = \omega^* t_0,\ \Omega' = \frac{\Omega}{\omega^*},\ \delta^2 = \frac{c_2^2}{c_1^2}$$
$$\varepsilon_T = \frac{\beta^2 T_0}{\rho C_e(\lambda+2\mu)}, \tag{3}$$

where $\omega^* = \frac{C_e(\lambda+2\mu)}{K}, c_1^2 = \frac{\lambda+2\mu}{\rho}, c_2^2 = \frac{\mu}{\rho}$.

Upon using quantities (3) in Eqs. (1) and (2), we obtain

$$\delta^2\nabla^2\vec{u} + (1-\delta^2)\nabla(\nabla.\vec{u}) - \nabla T = \ddot{\vec{u}} + \vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2(\vec{\Omega}\times\dot{\vec{u}}), \tag{4}$$

$$\nabla^2 T - (\dot{T} + t_0\ddot{T}) - \varepsilon_T\nabla.(\dot{\vec{u}} + t_0\ddot{\vec{u}}) = 0. \tag{5}$$

Here primes have been suppressed for convenience.

3. Dispersion equation and its solution

Let $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ be the rotating orthonormal basis and we take $\vec{\Omega} = \Omega\vec{e}_3$. A perturbation $\vec{u}$ that is collinear to $\vec{\Omega}$ is not affected by Coriolis or convective accelerations. Therefore, we limit the analysis to displacements in the plane $(\vec{e}_1, \vec{e}_2)$ which remain constant in the direction $\vec{e}_3$. On applying the successive application of divergence and curl operators to Eq. (4) leads to a system of following two coupled differential equations for e and $\vec{w}$ as

$$\left[\nabla^2 + \Omega^2 - \frac{\partial^2}{\partial t^2}\right]e - 2\Omega\dot{w}_3 - \nabla^2 T = 0, \tag{6}$$

$$\left(\delta^2\nabla^2 + \Omega^2 - \frac{\partial^2}{\partial t^2}\right)w_3 - 2\omega\dot{e} = 0, \tag{7}$$

where $e = \nabla.\vec{u}, \vec{w} = \nabla\times\vec{u} = w_3\vec{e}_3$. Clearly the longitudinal, transverse and thermal waves get coupled with each other here in contrast to wave propagation in an inertial medium.

We consider waves that propagate in the direction $\vec{e}_1$ of the form

$$(e, w_3, T) = (A_1, A_2, A_3)\exp\{\imath(kx_1 - \omega t)\},\quad \imath^2 = -1. \tag{8}$$

Upon using wave solution (8) in Eqs. (5)–(7), we obtain

$$[1 - v^2(1+\Gamma^{-2})]A_1 + 2\imath v^2\Gamma^{-1}A_2 - A_3 = 0, \tag{9}$$

$$2\imath v^2\Gamma^{-1}A_1 - [\delta^2 - v^2(1+\Gamma^{-2})]A_2 = 0, \tag{10}$$

$$\varepsilon_T\tau_0 v^2 A_1 - (1 - \tau_0 v^2)A_3 = 0, \tag{11}$$

where $\tau_0 = \imath\omega^{-1} + t_0, v = \frac{\omega}{k}$. Here $\Gamma = \frac{\omega}{\Omega}$ is called Kibel number.

The condition for the existence of non-trivial solution for A_1, A_2 and A_3 of system of Eqs. (9)–(11) yields the dispersion equation

$$\prod_{i=1}^{3}(1 - v^2\xi_i^2) = 0. \tag{12}$$