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Dynamic modeling of parallel robots with flexible platforms

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ABSTRACT

This paper presents a method for calculating the direct and inverse dynamic models of a parallel robot with a flexible platform. The system considered in this study is a Gough–Stewart 6-DOF parallel robot however the method is general and can be used for other structures. The platform of the parallel manipulator is considered as a flexible body and modeled using distributed flexibility while the links of the legs are considered as rigid. The direct dynamic model gives the elastic and Cartesian accelerations in terms of the input torques and the current state of the system i.e. the position and velocities of both the rigid and elastic variables. The inverse dynamic model calculates the elastic accelerations and the actuator torques from the current state variables and the desired acceleration of the platform.

1. Introduction

The dynamic modeling of Gough–Stewart robot with rigid elements has attracted many works with different algorithms. For instance, the Lagrange–Euler formalism has been used in the works of Lee and Shah [1], Geng et al. [2] and Lebret et al. [3], Ait-Ahmed [4], Bhattacharya et al. [5,6] and Liu et al. [7]. The principle of virtual work has been used by Tsai [8], Codourey [9] and Staicu [10,11]. On the other hand, Newton–Euler equations have been used in the work of Sugimoto [12], Reboulet et al. [13], Ji [14], Gosselin [15] and Dasgupta et al. [16,17]. However, recently, Carricato and Gosselin [18], Afroun et al. [19], Fu et al. [20] and Vakil et al. [21], have pointed out common errors in many methods related to parameterization and instantaneous kinematic behavior of the legs. These errors may cause kinematic and dynamic miscalculations. The correct dynamic modeling of the rigid Gough–Stewart robot, which avoids these errors, has been demonstrated using different formalisms. For example using screw theory in Gallardo et al. [22], the Newton–Euler approach in Khalil and Guegan [23], Khalil and Ouarda [24] and by Lagrange methods in Abdellatif and Heimann [25].

The aim of this paper is to extend the dynamic method in [23,24] to parallel robots with flexible platforms. There are two possible applications for this work. The first application is for robots with large platforms, where flexibility can no longer be neglected. The platform's flexibility can be taken into account in the design of the controller, thanks to this model. The second application is for robots that carry out high speed machining tasks, during which large vibrations are induced. Generally to counteract this, the platform's mass is increased until the effects of vibration are negligible. This solution leads to manipulators with high mass and greater energy consumption.

To give an idea of the dimension involved, consider CMW's 6-DOF parallel robot the hexapode. The platform of this robot has a mass of over 200 kg with a diameter of 600 mm. The total mass of the system is 900 kg. The maximum speed is just over 0.8 m/s. If the flexibility is modeled, these manipulators can be designed with low weight platforms, thereby reducing the total mass and permitting the use of high acceleration trajectories.

In the literature the main approaches to modeling flexibility in parallel robots are concerned with limb flexibility, this is because the limb's flexibility can be approximated using beam elements. For instance, for the Gough–Stewart robot the effects of leg flexibility

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Nomenclature

$\mathbf{q}_i$	Joint positions of leg i
$\dot{\mathbf{q}}_i$	Joint velocities of leg i
$\ddot{\mathbf{q}}_i$	Joint accelerations of leg i
Γ_i	Joint accelerations of leg i
$\mathbf{q}_e$	Generalized elastic position variables of platform
$\dot{\mathbf{q}}_e$	Generalized elastic velocity variables of platform
$\ddot{\mathbf{q}}_e$	Generalized elastic acceleration variables of platform
$\Phi_{dk}(\mathrm{i})$	Displacement shape function of mode k at point i
$\Phi_{rk}(\mathrm{i})$	Rotation shape function of mode k at point i
$\mathbf{V}_i$	Kinematic twist at point i 30
$\mathbf{v}_i$	Linear velocity at point i
ω_i	Angular velocity at point i
$\mathbf{r}_i$	Vector from platform origin to point i
$\mathbf{F}_i$	Wrench at point i
$\mathbf{f}_i$	Force at point i
$\mathbf{n}_i$	Moment at point i
$\mathbf{Q}_p$	Elastic generalized forces of the platform

are examined in [26,27]. The optimum choice of flexibility representation is investigated in [28]. In [29,30] lumped spring mass approximations have been used. By using distributed flexibility in [31], a solution for the dynamics calculation of parallel robots is proposed in the case of flexible legs but with a rigid platform.

The parallel robot treated in this paper is the well known Gough–Stewart platform, which is considered as a good representation of parallel robot's characteristics, however the proposed methods are general and can be applied to other parallel robots. The paper is organized in the following way. In Section 2, the overall procedure is outlined, as well as the prescribed solution. In Section 3 the geometric, kinematic and dynamic models of the manipulator legs are presented. In Section 4, the generalized Newton–Euler model of a flexible platform is given. Furthermore the geometric and flexible parameters for the target platform are described. Section 5 describes how the inverse dynamic model and direct dynamic model of the flexible robot are derived. In Section 6, a numerical simulation validating the proposed model is given. Finally in Section 7 the conclusions are drawn and future areas of research are described.

2. Problem statement

The objective of this work is to calculate the dynamic models of the Gough–Stewart robot with flexible platform. The inverse dynamic model obtains the joint torques and forces for a desired acceleration of the platform using the state variables of the robot (the positions and velocities). The direct dynamic model gives the elastic and rigid accelerations of the system's variables in terms of the input torques and the state of the system.

In order to proceed, the system is decomposed into two subsystems, one is flexible and the other is rigid. The decomposition is performed by opening (virtually) the spherical joints representing the connection points between the legs and the platform. The flexible subsystem represents the platform, that is described using distributed flexibility [32,33] and modeled using Cartesian coordinates and the Newton–Euler formulation. The rigid elements of the robot, which consist of the legs and the fixed base of the legs, are described as a tree structure robot using the Modified Denavit Hartenberg Parameters [34] and modeled using joint variables. The two subsystems are connected by calculating the reaction forces at the connection points between the platform and the legs.

3. Leg system description and modeling

3.1. Geometric parameters

The studied system is a Gough–Stewart structure, as shown in Fig. 1. The platform has 6-DOF and is connected to the fixed base by six legs. Each leg is connected to the base with a 2-DOF universal joint (U-joint) and to the platform with a 3-DOF spherical joint (S-joint). Each leg has a variable length by means of an actuated prismatic joint (P-joint).

The base frame and the platform frame are denoted by Σ_o and Σ_p, respectively. The connection points between the base and the U-Joints are denoted as $\mathbf{b}_i$ and are arranged according to the convention established in [23]. The connection points between the platform origin and the legs are denoted as $\mathbf{p}_i$, for $i = 1 \ldots 6$.

After opening virtually the spherical joints, each leg i is composed of three joints and three links. The geometric parameters of the links, $j = 1 \ldots 3$, for each leg i are given in Table 1, for $i = 1 \ldots 6$.