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Calculation of ball bearing speed-varying stiffness☆

Xia Sheng, Beizhi Li *, Zhouping Wu, Huyan Li

Center of Advanced Manufacturing Technology, School of Mechanical Engineering, Donghua University, North Renmin Road, Songjiang District, Shanghai, China 201620

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ABSTRACT

Dynamic properties of ball bearing (or any rolling bearing) are varying during operation, especially the stiffness which mostly depends on rotating speed and loads applied. Therefore, in this paper, the notion of rolling bearing speed-varying stiffnesses introduced and explained by studying the relations of load-deflection through the bearing dynamic model which is based on Jones & Harris's efforts. After the acquisition of load-deflection relations, two common numerical (linearization) methods are provided to calculate bearing stiffness, which turned out to be too unstable to get satisfied results. To improve the method of calculating speed-varying stiffness, an analytic model based on the differentiation of implicit function is proposed to calculate the speed-varying stiffness. The comparisons between the results from the numerical method and the proposed method have been made to validate the proposed method. In addition, the results calculated by the proposed method are compared with those which appeared in other literatures or experiments.

1. Introduction

Rolling bearings can be seen in all kinds of rotational machineries for their low-cost, high precision and high reliability. As support parts of a rotating system, the rolling bearings play a crucial role in the transmission of the loads and vibrations. Thus, it is needed to study the dynamic properties of the rolling bearing, particularly the high speed rolling bearing. When a bearing is operating at high speed, centrifugal and gyroscopic effects induced by the rotating rolling elements in the bearing will have great influence on the entire dynamic properties of the bearing, including the bearing stiffness which is very important to the analysis of dynamic behavior of the rotor-bearing system. Most previous research focused on the static bearing stiffness, considering it a constant and adopting it in dynamic analysis. For a lower speed rotor-bearing system, using static bearing stiffness in dynamic analysis may have a conclusion close to the real one. However in a high speed situation, bearing stiffness will change a lot due to the effect of speed and variations of bearing internal load distribution.

Ball bearings, as a kind of rolling bearing, are one of the most important parts of spindle-bearing system. Many researchers [1–3] found that spindle performance changes dynamically due to the nonlinear effect of stiffness of the bearings. M. Matsubara [4] studied this nonlinear effect and proposed a piecewise-linear model to simulate the nonlinearity of bearing stiffness. But Matsubara's paper didn't include the influence of speed on bearings. J. Jedrzejewski [5] gave a view about relations between bearing running speed and axial stiffness. It was found that axial stiffness would continue dropping if the rotational speed grew. Matti Rantatalo et al. [6] had the same point, they predicted that the speed-varying radial stiffness would fall down to 40% of the static radial stiffness when the

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* Corresponding author.
E-mail addresses: warterfly@gmail.com (X. Sheng), lbzhi@dhu.edu.cn (B. Li).

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spindle runs at 20,000 rpm. However, both of them didn't give a detailed model of the calculation of speed-varying stiffness which should depend on the analysis of bearing dynamic characteristics.

Early studies on rolling bearing dynamic characteristics were started by A.B. Jones [7,8] who studied the ball motion and internal contact mechanism and their relations. Later on, T.A. Harris reviewed Jones's theory and further concluded a model which was suitable for the calculation of bearing internal load distribution and bearing deflection [9,10], which was widely known as Jones & Harris model. Although Jones & Harris model did well in studying the dynamic characteristics in rolling bearings, their equations were complex in calculation. There were some other meaningful studies concentrated on the bearing deflection vs. applied load [11,12], though they were analyzed or measured statically. Gargiulo [13] formed empirical formulae for the load-deflection displacement relations by assuming rigid bearing races, whose conclusion was regarded as a benchmark for bearing stiffness. It is available to calculate bearing stiffness based on the theories listed above, but these theories either were proposed in static state or they cannot determine tilting and cross-coupling stiffness between the radial, axial, and tilting deflections of bearings which are necessary to form the stiffness matrix.

T.C. Lim and R. Singh [14] proposed a comprehensive bearing stiffness matrix of six dimensions which demonstrated a coupling between the shaft bending motion and the flexural motion on the casing plate. Hoon–Voon Liew [15] analyzed time-varying rolling element bearing characteristics based on T.C. Lim's bearing stiffness model. Yi Guo [16] created a FEM model to numerically calculate the bearing stiffness matrix which covered a wide range of bearing types and was also in six dimensions, this method, however, was a time-consuming job as the numerical results were obtained by FEM which would certainly take much time. Besides, their models were on the assumptions that didn't consider the dynamic behaviors in bearings. That means the models proposed above didn't realized influence of rotational speed on bearing stiffness. G.D. Hagiu [17] developed a theoretical research concerning the rigidity and damping characteristics of high speed angular contact ball bearings. It studied deformations of bearings by adopting Hertzian contact theory, but the definition of bearing stiffness was not referred.

Many researchers also estimated bearing stiffness by carrying out the experiments. Walford and Stone [18] developed a rotor-bearing test rig to estimate the bearing's radial stiffness and damping by measuring the response of the rotor. It also studied the effect of temperature and rotating speed on the stiffness of the bearings. Then Stone published a review of the measurements of bearing stiffness [19]. In his review, various kinds of method in measuring the stiffness were introduced. Similarly, Kraus et al. [20] measured stiffness and damping characteristics of a radial ball bearing by performing experimental modal tests on a transmission test stand. Their results indicated that static bearing stiffness is very close to the stiffness measured when the bearing is running. However their conclusion is not convincing enough to prove that speed-varying stiffness can be estimated by static stiffness because their testing speed was only up to 1000 rpm which now seems to be a very low rotational speed. Rajiv Tiwari and Nalinaksh S. Vyas [21] performed a series of experiments on a multi-disk rotor-bearing system to estimate the bearing stiffness through examining random responses. In this research, they measured the rotor displacement and velocity signals and set up a relation between rotor displacement and bearing stiffness rather than the relation between bearing displacement and bearing stiffness.

In this paper, a mathematical model for calculating bearing speed-varying stiffness is proposed by analyzing the equations in the bearing dynamic model which is based on Jones & Harris's efforts [10]. This mathematical model aims to give a comprehensive consideration on the nonlinear stiffness of ball bearing and the influence of speed on the bearing stiffness that affects the load vs. deflection displacement characteristic of bearing which also affects the dynamics of ball bearing-supported rotor system. Extending Jones & Harris's model to an analytical system of five degrees of freedom, it also provides a stiffness matrix of five dimensions. Moreover, the comparisons have been made with other views on bearing stiffness to see how the effect of speed affects bearing stiffness.

2. Starting assumption

Some assumptions are necessary due to the complicated interactions between bearing elements and parts. These complexities include indeterminate bearing internal geometry, nonlinear interaction between the rolling bearing elements and the motions of internal rolling elements. Therefore, to establish a practical mathematical model, the following assumptions are proposed:

- The bearing analyzed is ideally shaped and is free from faults.
- Bearing outer ring is fixed in space while the inner ring will deflect in radial or axial direction.
- The outer ring and inner ring are assumed not to be bent. Small deformations occurring at the contact areas of rolling elements and raceways do not affect the entire shape of both rings.
- Rolling elements are separated by a constant angle by the cage. The dynamic behavior of the cage is neglected.
- The bearing operates under isothermal conditions.
- The external loads act centrically upon the rings, which will not cause angular misalignment of the bearing rings.
- Tribological issues are beyond the scope of this study, thus, lubrication is ignored.

These assumptions above are generally accepted in most analyses of rolling bearings. Some further assumptions will be introduced and discussed to serve this work in special cases.