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Free vibration analysis of a plate on foundation with completely free boundary by finite integral transform method

Yang Zhong [a,*], Jian-Hua Yin [b]

[a] *Department of Civil Engineering, Dalian University of Technology, Dalian 116024, PR China*
[b] *Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong*

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Abstract

The theoretical solutions of eigenfrequencies and vibration modes of a rectangular thin plate on an elastic foundation with completely free boundary are derived by using a double finite cosine integral transform method. In the analysis procedure, the elastic foundation is regarded as a Winkler elastic foundation model. Because the basic dynamic elasticity equations of the thin plate on elastic foundation are only used, it is not needed to select the deformation function arbitrarily. Therefore, the solution developed in the present paper is more reasonable and more accurate. To prove the correctness of the solutions, numerical results obtained using the present solutions are compared with those in the literatures.

Keywords: Elastic foundation; Rectangular thin plate; Free boundary; Eigenfrequencies; Vibration modes; Finite cosine integral transform

1. Introduction

The use of thin plates is common in all fields of structural, civil, and mechanical engineering and aerospace. The vibrations of rectangular plates with various boundary conditions have been extensively investigated for many years. The related publications can be counted in thousands (Leissa, 1993). In addition, a literature survey reveals that most previous investigations have largely dealt with a scheme or technique that is only suitable for a particular type of boundary conditions. Due to the mathematical complexity of the situation, it is well known that the analytical solutions are generally available only for plates that are simply supported along at least one pair of opposite edges. Leissa gave a survey of research on rectangular plate problems up to 1970 (Warburton, 1954; Leissa, 1973). The a further overview up to the beginning of this century is presented in references (Warburton, 1979; Warburton and Edney, 1984). One of the most commonly used methods in free vibration analysis of plates is the Rayleigh–Ritz energy technique, where appropriate functions associated with various boundary conditions are chosen to describe the lateral deflection of the deformed plates. The chosen functions almost always do not satisfy the governing differential equation. Gorman used the superpo-

* Corresponding author. Tel./fax: +86 411 84707837.
E-mail address: zhongyang58@163.net (Y. Zhong).

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sition technique to solve approximately free vibration problems of plates for various geometries and boundary conditions (Gorman, 1980, 1982). A set of static beam functions was used to determine the natural frequencies of elastically restrained plates (Bapat et al., 1988; Zhou, 1996). Hurlebaus et al. (2001) extended the Fourier series solution to other more complicated boundary conditions than the simply supported one. The other numerical approaches, such as finite element method (Yang, 1972) and boundary element method (Zafrang, 1995) were usually adopted by many researchers to analyze the plate on elastic foundation.

Integral transform is one of the best approaches to obtain the explicit solutions of some partial differential equations used in elasticity (Sneddon, 1972). This method has been often utilized to analyze some structural engineering problems (Sneddon, 1981). Unfortunately, based on the authors' knowledge, there are no reports about using finite integral transform to analyze a rectangular plate on an elastic foundation. In the design the highway rigid pavement or called cement concrete pavement is modeled as Kirchhoff thin plate with completely free boundaries. In airport pavement the higher order theories of plate (Mindlin Plate) are used.

In the paper, a double finite cosine integral transform method is adopted to acquire the theoretical solutions of eigenfrequencies and vibration modes for a rectangular thin plate with completely free boundary on an elastic foundation. In the analysis procedure, the elastic foundation is simplified as a Winkler elastic foundation. Because the basic dynamic elasticity equations of the thin plate on the elastic foundation are only used, it is not needed to select the deformation functions first and arbitrarily. Therefore, the solution developed by the present paper is more reasonable and theoretical. In order to prove the correctness of formulations, numerical results obtained using the present solutions are compared with those published in the literatures.

2. Vibration of thin plate on foundation and application of integral transform

According to the theory of the classical Kirchhoff plate, the governing equation of motion for an unloaded plate on the foundation is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + Kw(x, y, t) + \rho h/D\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where $K = k/D$ and $D = Eh^3/12(1 - v^2)$ is the flexural rigidity of plate. E, v, h, and ρ are the Young's modulus, Poisson's ratio, the thickness and the density of plate, respectively. The variable w is the out-of-plane displacement and k is the reaction coefficient of foundation. Assuming a harmonic vibration, one may write

$$w(x, y, t) = W(x, y)\sin\omega t \tag{2}$$

where $W(x, y)$ is the shape function describing the modes of the vibration and ω is the natural circular frequency of the plate. Substitution of Eq. (2) into Eq. (1) gives

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} + \lambda W = 0 \tag{3}$$

where $\lambda = K - \rho h\omega^2/D$

To solve the partial differential Eq. (3), the double finite cosine integral transform approach (Sneddon, 1981) is utilized.

If $f(x, y)$ is a function of the two independent variables x and y, defined on the square $0 < x < a, 0 < y < b$, the definition of double finite cosine integral transform is presented by the equation

$$\bar{f}(m, n) = \int_0^a \int_0^b f(x, y)\cos\alpha_m x\cos\beta_n y \mathrm{d}x\mathrm{d}y \tag{4}$$

The inversion formula of the double finite cosine integral transform can be derived as

$$f(x, y) = \frac{1}{ab}\bar{f}(0, 0) + \frac{2}{ab}\sum_{m=1}^{\infty}\bar{f}(m, 0)\cos\alpha_m x + \frac{2}{ab}\sum_{n=1}^{\infty}\bar{f}(0, n)\cos\beta_n y + \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\bar{f}(m, n)\cos\alpha_m x\cos\beta_n y \tag{5}$$

where $\alpha_m = m\pi/a$ and $\beta_n = n\pi/b \cdot a$ and b are the length and the width of the plate respectively.