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Computational structural analysis of planar multibody systems with lower and higher kinematic pairs

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ABSTRACT

AbstracKinematic and dynamic modeling of multibody systems requires an initial stage of topological recognition or structural analysis, in which the analyst must identify the model coordinates and a sufficient number of constraint equations to relate them. This initial phase could be solved quickly, safely and automatically, determining the kinematic structure of the multibody system; that is, dividing it into a set of kinematic chains called structural groups. Furthermore, structural groups are widely used for structural synthesis and so, the analysis and design of multibody systems can be integrated into the same software package. On the basis of known graph-analytical methods for structural analysis, a computational method that determines the kinematic structure of a multibody system from its adjacency matrix is developed and evaluated. This method allows the choosing of any type of coordinates (relative, natural or reference point) and the kinematic and dynamic formulations most appropriate for solving the problem. The algorithm has been applied to a large number of mechanical systems of different complexity, offering the same kinematic structure as was obtained through the application of graph-analytical methods.

1. Introduction

Structural analysis is the study of the nature of connection among the members of a mechanism and its mobility. It is concerned with the fundamental relationships between the degrees of freedom, the number of joints, the number of bodies and the type of joints used in a mechanism [1,2]. These relationships allow the solution of broad kinds of problems related to the kinematic structure of multibody systems: mobility analysis, structural synthesis and structural analysis. Structural synthesis is related to the design of mechanisms, and structural analysis is related to the derivation of their kinematic structure, that is, the decomposition of mechanisms into simple kinematic chains. A detailed discussion on the main research related to the kinematic structure of the mechanisms is presented in [3].

The theory of structural analysis was first introduced by L.V. Assur, in 1917, by establishing the methods that allow the decomposition of certain mechanisms into a set of kinematic chains called structural groups (SG).

Assur stated that any mechanism can be split into a set of kinematic chains called *Primary Elements* (PE) whose mobility equals the mobility of the whole system, and another set of kinematic chains with null mobility, known as *Assur Groups* (AG) [4]. Fig. 1 shows three mechanisms of different complexity and their decomposition into SG. In Fig. 1a a four bar linkage is depicted. This mechanism can split (Fig. 1b) into one PE (body {2}) and one AG with two bodies {3, 4}. The PE has one degree of freedom (DOF) and one input movement (θ_1, absolute rotation of body {2}). The mechanism in Fig. 1c splits into the same PE as in Fig. 1a and one AG of Class II (bodies {3–6} in Fig. 1d). However, mechanisms with higher kinematic pairs and relative input movements (Fig. 1e) cannot be decomposed into SG, PE or AG, applying Assur theory. The computational structural analysis of such mechanisms, based on existing

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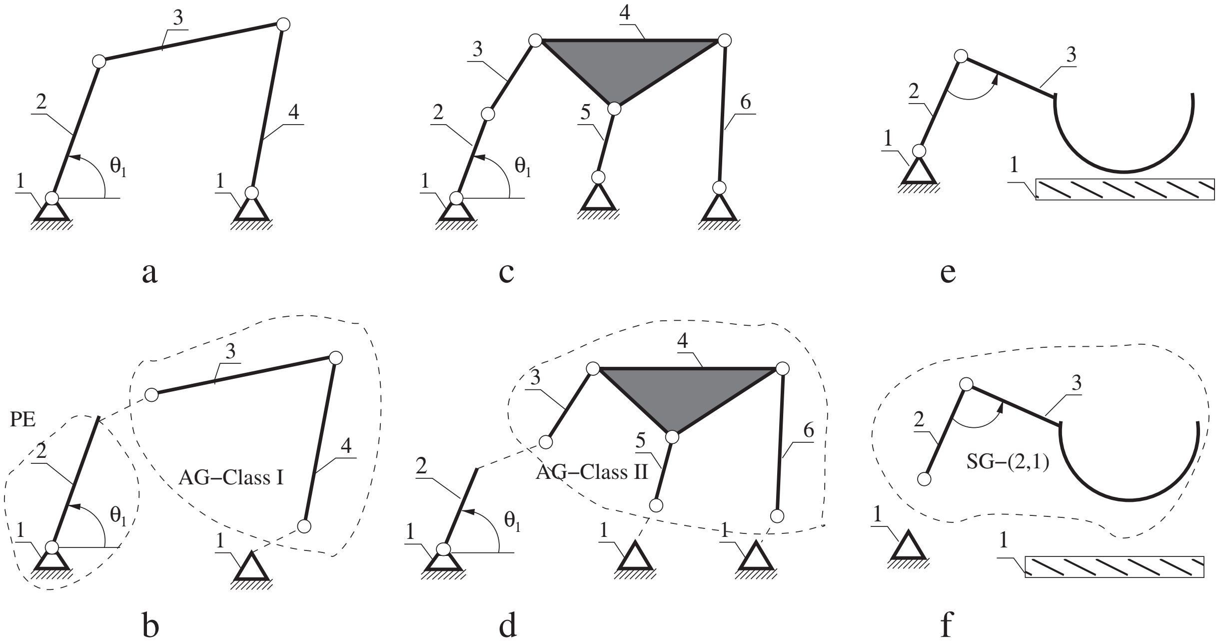

Fig. 1. Decomposition of mechanisms into SG. a) Four bar linkage. b) The four bar linkage splits into PE and one AG of Class I. c) A six body linkage. d) Decomposition of (c) into PE and one AG of Class II. e) Mechanism with higher kinematic pairs and one relative input movement. f) Decomposition of (e) into one SG formed by two bodies and one input movement.

graph-analytical methods [5] that expand the Assur theory for the structural analysis, is the main objective of the present study. Applying these methods to the mechanism in Fig. 1e only one SG formed by two bodies and one input movement is obtained (Fig. 1f).

Assur established the condition (null mobility) that any kinematic chain should be satisfied in order to become an AG, and classified them into different Classes (I, II, III, etc.) depending on the number of bodies that they contain (2, 4, 6, etc.). Nowadays, the concept of structural group is extended to any kinematic chain whose number of independent chain inputs n_c coincides with its mobility L_c,($n_c = L_c$).

The mobility L_c of kinematic chains and mechanisms can be determined by different methods. One of the well known methods is the Kutzbach–Grübler criterion which, applied to planar mechanisms, is shown in Eq. (1). Here, N_m is the number of mobile bodies and p_L, p_H stands for the number of lower and higher kinematic pairs, respectively.

$$L_c = 3 \cdot N_m - 2 \cdot p_L - p_H \tag{1}$$

This condition, known as the generation principle of mechanisms [5], allows both the decomposition of any mechanism into SG, which facilitates its kinematic and dynamic analysis, and the generation of any mechanism by simply adding SG to the frame or to other existing SG, which facilitates structural synthesis.

The application of SG decomposition to structural synthesis problems with both graph-analytical and computational methods based on Assur theory has been extensively treated in the literature for planar mechanisms [6–12] and to the design of gear trains [8,13,14]. The algorithm introduced in the present work could extend the structural synthesis of mechanisms which includes higher kinematic pairs and not only AG, but any kind of SG.

However, the use of methods based on SG decomposition applied to kinematic and dynamic analysis of multibody systems has not been deeply explored yet. In Saura [15], an algorithm for the kinematic computational analysis of linkages formed by any number of 3R Assur Groups (two body and three rotation kinematic pairs) is presented. The solution is recursively obtained starting from the SG data introduced by the user. The method is applied to solve a 10-body mechanism and the results are then compared to those offered by the use of commercial software and from the direct application of the kinematic formulation with the needed equations and Jacobians provided manually (specific purpose solution). In this reference [15] a number of advantages are enumerated and they motivate the present work.

Buskiewicz [16] introduces a computational algorithm for the structural analysis of planar mechanisms with one degree of freedom PE and one or more AG. The user has to introduce information regarding the number of loops, joints and moving bodies and the bodies and joints contained in the loops. The algorithm automatically identifies the AG and the elements in the Jacobian matrix of the whole system. Furthermore, the algorithm (SAM), optimizes the solution of the kinematic equations reordering the elements of the Jacobian matrix and offering a more efficient solution to the kinematic analysis. Though a qualitative improvement of the efficiency is not reported, it can be seen that reorganizing the null elements of the Jacobian should provide a reduced time analysis.