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Sensitivity analysis of parallel manipulators using an interval linearization method

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ABSTRACT

The subject of this paper is about an interval linearization method for the sensitivity analysis of manipulators to variations in their geometric parameters. First, the proposed method is presented. Then, three manipulators are used as illustrative examples: The five-bar mechanism, the 3-RRR planar parallel manipulator and the Orthoglide. The benefits and restrictions of the proposed method are also discussed and appropriate indices are derived to show the efficiency of the method. The obtained results are also compared with the results obtained with frequently used methods. The proposed method is simple to implement and provides verified results in low computational time and thus can be applied to complex robots such as the Orthoglide. In particular, the standard linearization method computes unreliable results near singularities, whereas the proposed interval linearization method automatically detects such situations.

1. Introduction

Many robotic applications require high precision. This precision cannot be achieved unless all sources of errors have been identified and integrated in dedicated computational methods. Amongst the well known sources of errors we find manufacturing errors, joint clearances, and backlashes in the actuators. The sensitivity analysis of a manipulator aims at knowing the influence of variations in its geometric parameters and/or actuators on its performance. This information is useful for the evaluation of the pose error of the end-effector and for the tolerance synthesis. The three main methods used for the sensitivity analysis to variations in geometric parameters and joint clearances of serial and parallel robots are: (*i*) the probabilistic methods [1]; (*ii*) the linearization methods [2]; and (*iii*) the interval analysis methods [3].

As indicated in [4], science was based in the last centuries on deterministic ideas, which consist in believing that since each phenomenon is due to a determined cause, the behavior can be predicted. However, since it is usually impossible to gather enough information for the behavior to be predictable, probabilistic theories were born. The probabilistic approaches are well known in several engineering fields such as nonlinear dynamics and robotics. Recently, in [5], a probabilistic approach was used in the field of Human–Robot communication. In [1], the sensitivity analysis for a two-link planar manipulator and the Stanford arm was conducted according to a probabilistic method. The authors pointed out that the probabilistic model of the kinematics and dynamics of these robots will be reliable if a sufficient number of experiments are conducted, which means that the results will not be verified. The probabilistic approaches present the advantage of having a competitive computational time while being simple and general. However, they do not provide any verified result.

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Other methods for sensitivity analysis do exist such as the *linearization method*, which consists in analyzing the sensitivity of the linearized system. The linearization method is a numerical method that does not provide verified results [6–8]. In [2,9–11], the authors worked on the sensitivity analysis of planar parallel manipulators using the linearization method. This method is suitable for small uncertainties and as long as the robot is not close to singular configurations. The computed sensitivity may be lower than the real one. For instance, the linearization method was used to analyze the sensitivity of the Orthoglide to variations in its geometric parameters [12].

The sensitivity of parallel manipulators to joint clearances is also an important issue. The great impact that joint clearances can have on the end-effector pose of the manipulator was highlighted in [8]. Furthermore, as mentioned in [13], errors due to joint clearances cannot be compensated by calibration contrary to manufacturing and assembling errors. To analyze the effect of joint clearances, a complete mathematical model of the joint should be defined. For instance, a joint can be modeled as a journal bearing [14]. The same model was used in [15] in order to compute the effect of joint clearances. A general approach was proposed in [16] but turns out to be time consuming. A general error-prediction model has been recently developed in [17]. Accordingly, two optimization problems were formulated in order to find the maximum positioning and orientation errors of the end-effector of the manipulator due to joint clearances. Again, the previous research works without minimizing their importance are based on algebraic methods that do not provide verified results.

On the contrary, interval analysis based approaches provide verified results while being simpler than their counterparts. In [18] the authors used a Newton–Raphson based interval analysis method combined with a bisection algorithm in order to analyze the sensitivity of simple mechanisms to joint clearances. However, their approach is complicated and time consuming because it requires a bisection algorithm. Moreover, the method has not been applied to complex mechanisms, and the authors did not give enough information about the behavior of their algorithm close to singular configurations.

The standard linearization method turns out to be simple to use and provides good results as long as the variations in the parameters are small enough. Therefore, the standard linearization method may provide bad results.

In this paper, we propose an interval linearization method that combines the simplicity of the standard linearization method and the verification of the results. Moreover, the proposed method is simpler than the approach described in [18] and allows us to analyze the sensitivity of complex robots to geometric errors and joint clearances.

The paper is organized as follows. Section 2 introduces the proposed interval linearization method for the sensitivity analysis of parallel manipulators. An efficiency index of the method is also developed with regard to the amount of uncertainties. Section 3 deals with the sensitivity analysis of the five-bar mechanism and makes a comparison between the results obtained with the proposed method and the ones obtained with a standard linearization method. Section 4 deals with the sensitivity analysis of the 3-RRR planar parallel manipulator with the proposed interval linearization method. Section 5 deals with the sensitivity analysis of the Orthoglide, a three degree of freedom translational parallel manipulator, using the proposed interval linearization method and two geometric models of the manipulator.

2. The interval linearization for sensitivity analysis

2.1. Interval analysis

Intervals are denoted by brackets: Intervals of reals are $[x] = [\underline{x}, \overline{x}]$ with $\underline{x}, \overline{x} \in \mathbb{R}$ and $\underline{x} \leq \overline{x}$. Intervals of vectors (also called boxes) are $[\mathbf{x}] = [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$ with $\underline{\mathbf{x}}, \overline{\mathbf{x}} \in \mathbb{R}^n$ and $\underline{\mathbf{x}} \leq \overline{\mathbf{x}}$ (the inequality being defined component wise); in this case the interval $[x_i] = [\underline{x}_i, \overline{x}_i]$ is the ith component of the interval vector. Interval of matrices are defined similarly by $[\mathbf{A}] = \left[\underline{\mathbf{A}}, \overline{\mathbf{A}}\right]$ with $\underline{\mathbf{A}}, \overline{\mathbf{A}} \in \mathbb{R}^{n\tilde{n}m}$ and $\underline{\mathbf{A}} \leq \overline{\mathbf{A}}$.

Interval analysis extends usual real operations (like scalar addition, multiplication, matrix/vector additions and multiplications) to interval objects. These interval operations are defined and implemented so as to rigorously verify the following containment property: The result of the interval operation contains all possible results of the corresponding real operation for real arguments inside the interval arguments. For example, $[x][y] + [x] \supseteq \{xy + x : x \in [x], y \in [y]\}$. Matrix and vector operations are extended in the same way, e.g. $[\mathbf{A}][\mathbf{x}] \supseteq \{\mathbf{Ax} : \mathbf{A} \in [\mathbf{A}], \mathbf{x} \in [\mathbf{x}]\}$, or $[\mathbf{A}] + [\mathbf{B}] \supseteq \{\mathbf{A} + \mathbf{B} : \mathbf{A} \in [\mathbf{A}], \mathbf{B} \in [\mathbf{B}]\}$. See [19] and references therein for more details. These interval operations are implemented in many environments, we used the Matlab library `Intlab` [20].

The interval evaluation of an expression generally provides a pessimistic enclosure of the range, which is a central issue in interval analysis. The so called mean-value extension can provide a tighter enclosure of the range, in particular when interval arguments have small widths. It uses interval evaluations of the function derivatives in order to enclose the function's graph within an interval linearization, whose range contains the range of the original function. More formally,

$$\{f(\mathbf{x}) : \mathbf{x} \in [\mathbf{x}]\} \subseteq f(\widetilde{\mathbf{x}}) + \sum_i \left[f_{x_i}\right]([\mathbf{x}])([x_i] - \widetilde{x}_i), \tag{1}$$

where $\widetilde{x} \in [x]$, usually chosen as its midpoint, and $\left[f_{x_i}\right]$ is an interval extension of the partial derivative of f with respect to x_i, which can be computed either by the interval evaluation of its derivatives or by using interval automatic differentiation.