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A plane symmetric 6R foldable ring

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ABSTRACT

The design of a deployable structure which deploys from a compact bundle of six parallel bars to a rectangular ring is considered. The structure is a plane symmetric Bricard linkage. The internal mechanism is described in terms of its Denavit–Hartenberg parameters; the nature of its single degree of freedom is examined in detail by determining the exact structure of the system of equations governing its movement; a range of design parameters for building feasible mechanisms is determined numerically; and polynomial continuation is used to design rings with certain specified desirable properties.

1. Introduction

Several different types of foldable frames, which in their deployed configurations form (often regular) polygons, have appeared in literature in the past 40 years. Bennett or Bricard [1] linkages are frequently used as the basis of linkages for foldable deployable frames. An early example appears in [2], in which an even number of bars are linked together in such a way that they can be folded into a tight bundle, and unfolded to form a regular polygon. In the six bar case, a three-fold symmetric linkage results [3], while in the four bar case, a Bennett linkage is formed [4,5]. Four bar foldable frames have been extensively examined [6,4,7]. For the six-bar case, Wohlhart [8,9] and Racilla [10] have focussed on the kinematics of the trihedral, or 'rectangular' member of the Bricard family. In [11], Pellegrino et al. proposed a new family of six bar foldable frames. A two-fold symmetric member of this family has been proposed as a support for a solar blanket, and its kinematics examined numerically [12,13]. Recently, the two-fold symmetric 6R foldable frame was identified as a special line *and* plane symmetric Bricard linkage [14]. This particular variant does, however, suffer from problems with bifurcations (although certain designs avoid this). If one of the two planes of symmetry is removed, a mobile 6R ring which experiences fewer problems with bifurcations remains. An example is shown in Figs. 1 and 2. In this paper, a greater understanding of the plane symmetric 6R foldable ring is sought by first identifying the ring as a plane symmetric Bricard linkage, examining the nature of its mobility using a cascade of homotopies [15] to identify positive dimensional solution sets (an application of polynomial continuation), determining a range of design parameters for building feasible mechanisms of this type, deriving a closed form expression for the linkage's kinematics, and finally employing polynomial continuation, again, in an attempt to design a family of plane symmetric 6R foldable rings with certain desirable practical properties.

2. Linkage specification

The two-fold symmetric ring of [14] has two different bar lengths (l_1 and l_2), and the bars are all tilted from the vertical by a single angle μ. Square cross-sectioned, prismatic (i.e., untwisted) bars were used. By contrast, the 6R linkage considered here has

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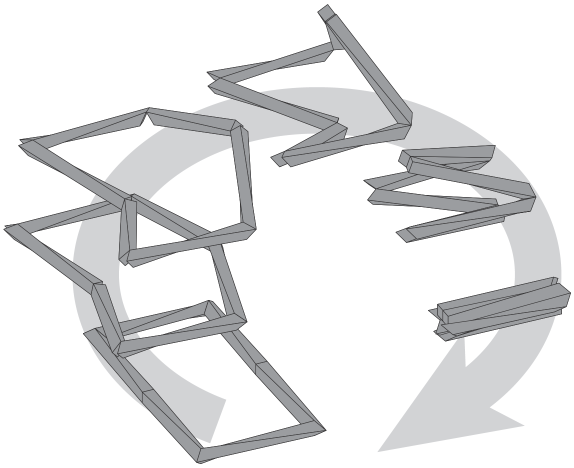

Fig. 1. Folding process for a linkage with parameters $\alpha_1 = \pi/4$, $\alpha_2 = -\pi/4$ and $\gamma = \pi/2$. Each bar is shown as a twisted prismatic bar with square cross-section.

all six bar lengths the same (l), and four separate bar tilt angles ($\alpha_1', \alpha_2', \beta_1$ and β_2), introducing a requirement that, if the bars have a square cross-section, some of the bars must be twisted in order to match the prescribed tilt angles at each end. A simplified diagram of the deployed linkage with all design parameters labelled is given in Fig. 3, while a representation of the physical linkage is given in Fig. 4, in which the twists in the bars are clearly visible. While a square cross-section is not required to construct the linkage, it does aid in visualising the bar twist.

There are six hinges (labelled $\mathbf{h}_1 - \mathbf{h}_6$), each with a single rotational degree of freedom connecting the bars in a closed loop. The plane of symmetry is preserved through the folding motion. It is labelled as the *XZ* plane in the fully deployed/open configuration, shown in Fig. 3. The plane contains the points $\mathbf{p}_6$ and $\mathbf{p}_3$, and the vectors $\mathbf{h}_6$ and $\mathbf{h}_3$, which are inclined to the *Z* axis by angles β_1 and β_2 respectively. Also when deployed, hinges $\mathbf{h}_1$ and $\mathbf{h}_2$ lie in planes rotated from the *YZ* plane by 45° about the *Z* axis. The angles these hinges form to the horizontal can be specified in two important ways. When constructing physical models of the plane-symmetric 6-bar, the most intuitive form is obtained by taking the projection of the hinges onto the *YZ* plane, and considering the angle formed between that projection and the *XY* plane, labelled here as α_1' and α_2'. This projection is shown in Fig. 3. This is a more intuitive definition as it specifies the tilt angle that a square cross-sectioned bar would need to form with the horizontal before cuts at 45° are made. However, future mathematical results are simplified by directly taking the angle between the *XY* plane and the hinge vectors, written as α_1 and α_2 here. Relationships between the two α definitions can be constructed as:

$$\begin{aligned}
&\tan\alpha = \frac{1}{\sqrt{2}}\tan\alpha' \\
&\Rightarrow \sin\alpha = \frac{\sin\alpha'}{\sqrt{\sin^2\alpha' + 2\cos^2\alpha'}} \\
&\Rightarrow \sin^2\alpha = \frac{\sin^2\alpha'}{1+\cos^2\alpha'} \\
&\cos^2\alpha = \frac{2\cos^2\alpha'}{1+\cos^2\alpha'}.
\end{aligned}$$

In order for all the bars to be parallel when fully stowed, the hinge vectors $\mathbf{h}_6$ and $\mathbf{h}_3$ must also be parallel in the fully folded configuration since they must be perpendicular to both the plane of symmetry and the ends of the bars to which they are

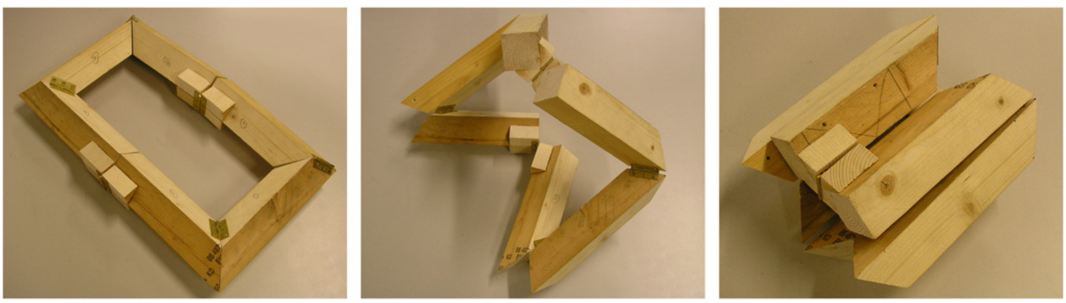

Fig. 2. The folding of a wooden model of the linkage shown in Fig. 1.