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Discussion of the dynamic stability of a multi-degree-of-freedom rotor system affected by a transverse crack

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ABSTRACT

The dynamic behaviour of cracked rotors is one of the most discussed topics in the rotordynamic literature due to the wide range of problems that may arise from this fault. Among them, it is a common notion that cracks in horizontal rotating shafts may cause instability of the system because of the periodic opening and closing of the crack, i.e., the *breathing mechanism*, determines the stiffness variation and the parametric excitation of the rotor system. Simplified models have been used to study this phenomenon using Jeffcott rotors. For the first time in this paper, a model of a real hyperstatic rotor with several degrees of freedom is used, which also considers the bearings and the foundation of the system, and the stability is discussed by means of the Floquet theory. The sensitivity of the obtained results to the system anisotropy and the crack position is also investigated. The results presented are quite different from those obtained by means of the simple Jeffcott rotor but are consistent with real and documented field experiences.

1. Introduction

The shafts in rotating machinery used in industrial plants are subjected to heavy working conditions that also include thermal transients. In spite of proper design and accurate manufacturing, the rotor shafts may be affected by fatigue cracks [1]. These cracks generally originate from manufacturing flaws, but their growth is associated with cyclic loading. Because these cracks may lead to catastrophic failures [2], their early detection is desirable.

During rotation, a horizontal cracked shaft is characterised by a time-variant stiffness that depends on the position of the transverse crack with respect to gravity; that is, during one revolution, the crack opens and closes under the effect of the shaft weight [1]. This behaviour is known as the *breathing mechanism*, and the breathing associated with the stress distribution around the crack is responsible for the local variation in the shaft flexibility.

Since the 1950s, many different studies have appeared in the literature on the topic of cracked shafts, and a portion of them are related to the stability of the dynamic behaviour of cracked rotors.

One of the first papers on this subject was written by Gasch [3], who considered the dynamics of a cracked Jeffcott (or de Laval) rotor with two degrees of freedom (d.o.f.) and a hinge model for the breathing mechanism. The same author has further expanded this study over the last thirty years [4–6]. The dynamic behaviour of the Jeffcott cracked rotor has been investigated with respect to different rotational speeds, positions and depths of crack as well as with different types of rotor bearings. Gasch also proposed the use of *Floquet theory* to analyse the stability of the dynamic response and performed the so-called *Floquet analysis* (FA), which is based on the determination of the *transition matrix* eigenvalues. Gasch's results show unstable zones caused by cracks located at approximately $2/N \cdot \omega$, where ω is the natural frequency of the system, and N is an integer number. The predictions regarding the dynamic behaviour of Jeffcott cracked rotor have been experimentally verified by Muszyńska [7] using a test-rig that was able to accurately reproduce the behaviour of a Jeffcott rotor.

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Other authors in the literature have studied the effect of the breathing mechanism on the system stability; for instance, Huang et al. [8] determined the variation of the flexibility coefficients with respect to the shaft orientation (or the relative position of the crack during the revolution of the shaft). Sekhar studied the influence of one or two cracks on the rotor stability [9,10] by considering the real part of the eigenvalues of the linearised equation of motion. Yang et al. [11] focused their interest on the relationship between the chaotic response and the instability of the cracked rotors. Zhu et al. [12] considered the problem of avoiding instability in a cracked rotor supported by active magnetic bearings. Lin et al. [13] studied the effect of environmental variables on the cracked rotor stability. More recently, Chen et al. [14] and Sinou [15] offered further contributions to the discussion of cracked rotor stability.

Although the phenomenon of the opening and closing of the crack has been thoroughly discussed in the literature, up to the present, the stability of cracked rotors has been studied for simple models only. However, the behaviour of real rotors can be quite different from that of the Jeffcott rotor.

In this paper, a steam unit generator, which has developed a deep crack during its operating history [1,16,17] without showing any symptoms of instability during speed transients, is considered. For the first time, the authors show that this observed behaviour is completely contrary to the results predicted by the simple theoretical models based on Jeffcott rotors.

The aim of this study is to evaluate the stability of a rotor for different values of the rotational speed and the depth of crack by also considering the effects of anisotropy. Large calculation resources are necessary for this type of investigation, and this necessity may partially explain why this type of analysis has not been previously performed on a multi-degrees-of-freedom system.

The paper is organised as follows: in the two next sections, the generator model and the crack modelling are introduced. The main issues of Floquet theory related to the system stability are discussed in Section 4. In Section 5, selected algorithms for the transition matrix evaluation are proposed, and suitable modifications are introduced for managing the complexity of the considered system. The last section of the paper is devoted to a discussion of the stability results obtained by means of numerical calculations.

2. Rotor model

The majority of the literature on cracked rotors, apart from those related to the crack-breathing phenomenon, analyses the dynamic behaviour of the rotating machines by carrying out numerical simulations using a Jeffcott rotor with a crack in the disc position.

In these cases, the analytical models and the numerical simulations have shown that the system may become unstable for certain values of the rotational speed and of the crack depth. However, the Jeffcott model, which is useful for the introduction of basic problems, may be not representative of the actual system dynamics (cracked or un-cracked cases), especially in the case of strongly nonlinear systems.

To evaluate the response of actual systems when a crack appears and propagates, the implementation of a more complex model is required.

Among the parts that compose rotating machineries employed in power plants, generators are often affected by cracks. Several such cases in the last 50 years have been documented in [18–21]. Therefore, the theoretical stability of the dynamic behaviour of a generator, similar to that described in [1,16,17], is considered in this paper. In these cases, the cracked section reached ~50% of the cross-sectional area (Fig. 1) when the unit had operated for approximately 133,000 h. The crack growth proceeded over a period of seven years and approximately 150 start-ups, while only one year was sufficient for the growth of the last 25% of the cross-section area. This generator experienced notably high vibration levels, especially during the last few coast-downs before removal from service; however, the equipment did not display instability.

The portion of the complete machine taken into consideration in this work has length of 17 m and includes the exciter. The external diameters change depending on the position along the rotation axis, the internal diameter remains constant due to the hollow that exists to reduce the rotor weight and to allow the fitting of copper bars. The generator and the exciter are supported by three journal bearings; the first is located near the rigid coupling that connects the generator to the low-pressure turbine, and the others are located at the opposite rotor end.

As shown in Fig. 2, the model of the rotor is composed of 33 beam elements, and the bearings and pedestals are numbered from left to right. Each finite element has eight d.o.f.s (see Fig. 3); four of them are the displacements in the two directions orthogonal to the rotating axis, and the other four are the rotations around previous directions. The possible cracked elements that will be considered in the paper are shown as shaded in Fig. 2, and their positions correspond to those of cracks discovered on real generators (see [1]).

The effect of the oil-film forces in the journal bearings is modelled by means of linearised dynamic coefficients, and the foundation is modelled by means of pedestals.

All of the data shown in Figs. 4 and 5 are provided by the machine manufacturer for rotational speeds ranging from 300 rpm and 3000 rpm and were not recalculated in this work. Note that the pedestal coefficients for supports #1 and #2 are considered as speed-dependent and are tuned to reproduce the horizontal foundation resonance that occurred in the actual machine.

Different configurations were tested in terms of modelling to analyse the effects of the bearings and the pedestals on the stability. The four configurations tested are listed in Table 1.

In the first configuration, the dynamic coefficients of the bearings and the pedestals are considered to be independent of the rotational speed. Therefore, the stiffness and damping parameters are constant and are equal to the values assumed at 300 rpm.

In the second configuration, the effect of the bearings on the stability has been investigated; the bearing dynamic coefficients vary as shown in Fig. 4, whereas the foundation values are equal to those of configuration 1.