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System reliability analysis of the robotic manipulator with random joint clearances

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ABSTRACT

The paper presents an innovative method for computing reliability of a robotic manipulator such that the positional error in the entire trajectory remains within acceptable limits. This problem is equivalent to a series system reliability analysis that can be solved using the extreme value distribution of the positional error. The principle of maximum entropy is applied to derive this distribution. A novel feature of the analysis is the use of fractional moments, instead of integer moments commonly used in the entropy literature. The fractional moments are obtained from a small, simulated sample of positional error. Examples are presented to illustrate accuracy and efficiency of the proposed method.

1. Introduction

1.1. Background

Robot manipulators are widely used to improve the production efficiency and accuracy of repetitive processes in industries, such as automobile and aerospace manufacturing, various types of assembling plants and computer-aided medical surgery.

A manipulator consists of several rigid links and joints to transmit the motion or force from one point to another. The connections at the joints are not perfectly accurate due to the manufacturing tolerances, material deformation and wearing over the service life of the robot. The uncertain variations in dimensions of links and joint clearances affect the positional and directional control (or kinematics) of the motion performed by the robot. As a result, actual motion output of the manipulator deviates from the target output required by the design. This deviation between actual and intended positions of the end-effector is referred to as positional error in this paper. Other drawbacks of excessive joint clearances are erratic shocks, vibration and noise [1,2].

The kinematic reliability of a manipulator is defined as the probability that the manipulator realizes its required motion path or trajectory within a specified tolerance range [3]. The shape and size of specified range (or safety region) depend on the intended use of the manipulator. The reliability can be defined in two ways. The point reliability means that reliability is evaluated with reference to a particular point on the trajectory of the output motion, whereas the interval (or cumulative) reliability considers the reliability over a range of output motion [4].

In the context of the point reliability analysis, Kim et al. [5] applied the first-order reliability method (FORM) to compute the reliability of an open-loop manipulator with six degrees of freedom. Here, all geometric dimensions and joint angles were considered as normally distributed. Wang et al. [3] applied a hybrid dimension-reduction method to compute the mean and

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variance of the positional error. Based on the mean and variance, a normal distribution was fitted to the positional error. This approach was applied to compute the reliability of a slider-crank mechanism. By combining the Monte Carlo simulations with Kriging method, Lai and Duan [6] analyzed the reliability of a turning machine with a random coefficient of friction. Wu and Rao [7] discussed optimal allocation of tolerances to joint angles by modeling them as interval variables. Huang and Zhang [8] applied the Taguchi method to determine optimal specifications for a function generation mechanism under the constraints of positional accuracy and assembly cost.

The problem of interval or cumulative reliability is computationally intensive, because the failure region is represented by multiple performance functions. Literature in this area is rather limited. Zhang et al. [9] developed a stochastic process model of cumulative reliability. They modeled the upcrossings or downcrossings of a prescribed threshold by the positional error as the Poisson process. This approach was applied to analyze the reliability of a four-bar function generator mechanism.

1.2. Motivation and approach

Since the manipulator is designed as a mechanism path generator for repetitive work, it may be necessary in most cases that error is controlled in the entire trajectory, instead of a few points. This provides motivation for cumulative or system reliability of the manipulator. Approximate methods, such as FORM, are not adequate to analyze a large system reliability problem [5]. The first-order second moment (FOSM) method is of limited applicability, as it is based on a spurious assumption of the normality of the positional error.

Therefore, the key objective of this study is to develop a computationally efficient and accurate method for the cumulative (or system) reliability analysis of robotic manipulators. The developed method should be able to deal with a large number of implicit and correlated performance functions defining the system reliability. In this paper, the cumulative reliability analysis is formulated as a series system reliability analysis problem. The paper shows that the series system reliability is equivalent to the probability that the maximum positional error in the entire manipulator trajectory is less than a specified limit.

The distribution of maximum positional error is derived using the maximum entropy (MaxEnt) principle, widely used in probabilistic analysis. A novel feature of the paper is the use of fractional moments as constraints, instead of integer moments commonly used in the entropy literature. To compute fractional moments of the positional error, a small sample of positional error is simulated based on the kinematic model of the manipulator. The paper shows that proposed method is highly efficient, and it achieves the same accuracy as that obtained by a large scale Monte Carlo simulation method.

1.3. Organization

The manuscript has been organized as follows. Section 2 describes the kinematic model and joint clearance analysis of a six degrees of freedom elbow manipulator. Section 3 summarizes basic information for the reliability analysis of a manipulator. Section 4 presents a general derivation of the entropy-based probability distribution using fractional moments as constraints. An interesting point is that fractions are also determined through the entropy maximization method. Numerical results are presented in Section 5, and conclusions are summarized in the last section.

2. Model of a manipulator

The kinematic model of a manipulator is based on the equations of motion of a multi-body system. The joint clearance can be modeled by introducing additional degrees of freedom (DOF) or virtual links in the model. The sensitivity of the output motion to joint clearances can be computed through deterministic analysis of the kinematic model [10,11].

In this section, the kinematic analysis of a robotic manipulator is illustrated using an example of a six degrees of freedom (DOF) elbow manipulator taken from Kim et al. [5], and shown in Fig. 1. A similar example was presented by Wu and Rao [7]. Later, this example will be used to illustrate the reliability analysis method developed in this paper.

2.1. Kinematics

The kinematic equations of an open-chain manipulator are described by the Denavit–Hartenberg (D–H) matrix in terms of position and orientation as [12]

$$\mathbf{T}_{i-1}^{i} = \begin{bmatrix} \cos(\theta_i) & -\cos(\psi_i)\sin(\theta_i) & \sin(\psi_i)\cos(\theta_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\psi_i)\cos(\theta_i) & -\sin(\psi_i)\cos(\theta_i) & a_i\sin(\theta_i) \\ 0 & \sin(\psi_i) & \cos(\psi_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4} \tag{1}$$

where, α_i, d_i, ψ_i and θ_i are used to define the coordinates of an ith link as shown in Fig. 2. The coordinates of the positions of the end-effector can be written as a product of D–H matrices for all the links:

$$\mathbf{T}_{\text{End}} = \mathbf{T}_0^1 \times \mathbf{T}_1^2 \ldots \mathbf{T}_{l-1}^{l} = \begin{bmatrix} \mathbf{n} & \mathbf{s} & \mathbf{a} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$