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Effect of a characteristic length on crack spacing in a reinforced concrete bar under tension

T. Małecki, I. Marzec, J. Bobiński, J. Tejchman *

Gdansk University of Technology, Faculty of Civil and Environmental Engineering, 80-952 Gdansk-Wrzeszcz, Poland

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Abstract

The paper presents quasi-static FE-simulations of the crack formation in a reinforced concrete bar without stirrups subject to tension. The material was modeled with a continuum smeared crack model using an elasto-plastic constitutive law. A linear Rankine criterion with isotropic softening and associated flow rule was adopted in a tensile regime. To ensure the mesh-independency, the softening parameter was enhanced by a characteristic length of micro-structure by means of a non-local theory. Attention was laid to the effect of a different characteristic length of micro-structure and initial bond-slip stiffness on the spacing of localized zones.

Keywords: Bond-slip; Concrete; Localized zones; Elasto-plasticity; FEM; Non-local theory; Reinforcement

1. Introduction

The analysis of reinforced concrete elements is complex due to occurrence of strain localization in concrete. The strain localization which is a fundamental phenomenon in concrete under both quasi-static and dynamic conditions (Pijaudier-Cabot and Bazant, 1987; van Vliet and van Mier, 1996; Chen et al., 2001) can occur in the form of cracks (if cohesive properties are dominant) or shear zones (if frictional properties prevail). The determination of the width and spacing of strain localization is crucial to evaluate the material strength at peak and in the post-peak regime.

The goal of the research carried out at the Gdańsk University of Technology is to describe the crack formation in concrete and reinforced concrete elements using continuum (Bobinski and Tejchman, 2004, 2006) and discrete models (Kozicki and Tejchman, in press). In this paper, the results of a plane strain FE-analysis of primary cracks (width and spacing) in a reinforced concrete bar subject to tension in quasi-static with a continuum model are described. The calculations were carried out with an elasto-plastic constitutive law using a linear Rankine failure function. In the simulations, different characteristic lengths of micro-structure, reinforcement ratios and bonds between concrete and reinforcement were used.

* Corresponding author. Tel.: +48 58 347 14 81; fax: +48 58 347 26 96.
E-mail address: tejchmk@pg.gda.pl (J. Tejchman).

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2. Constitutive model for concrete

To describe the behaviour of concrete in a tensile regime during uniaxial tension, a Rankine criterion was used with a yield function f assuming isotropic softening defined as

$$f = \max\{\sigma_1, \sigma_2, \sigma_3\} - \sigma_t(\kappa), \tag{1}$$

where σ_i the principal stress, σ_t the tensile yield stress and κ the softening parameter (equal to the maximum principal plastic strain ε_1^p). The associated flow rule was assumed. The edges and vertex in the Rankine yield function were taken into account by an addition of plastic multipliers. To preserve the well-posedness of the boundary value problem, to obtain mesh-independent results and to include a characteristic length of micro-structure for simulations of a deterministic size effect, a non-local theory was used as a regularization technique (Pijaudier-Cabot and Bazant, 1987; Bazant and Jirasek, 2002). In the calculations, the softening parameter κ was assumed to be non-local:

$$\bar{\kappa}(\boldsymbol{x}) = \frac{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\kappa(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}}{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\,\mathrm{d}\boldsymbol{\xi}}, \tag{2}$$

where $\bar{\kappa}(\boldsymbol{x})$ is the non-local softening parameter, V the volume of the body, $\boldsymbol{x}$ the coordinates of the considered (actual) point, $\boldsymbol{\xi}$ the coordinates of the surrounding points and ω the weighting function. The chosen formula (Eq. (2)) satisfies the normalizing condition (Bazant and Jirasek, 2002). As a weighting function ω, a Gauss distribution function was used:

$$\omega(r) = \frac{1}{l_c\sqrt{\pi}} \mathrm{e}^{-\left(\frac{r}{l_c}\right)^2}, \tag{3}$$

where the parameter l_c is a characteristic length of micro-structure and r is a distance between two material points. The averaging in Eq. (3) is restricted to a small representative area around each material point (the influence of points at the distance of $r = 3l_c$ is only of 0.1%). The characteristic length is related to the micro-structure of the material (e.g. maximum aggregate size and spacing in concrete, Bazant and Jirasek, 2002). It is usually determined with an inverse identification process of experimental data (Geers et al., 1996). However, the determination of one representative characteristic length of micro-structure l_c is very complex in concrete since strain localization can include a mixed mode (cracks and shear zones) and the characteristic length (which is one-dimensional) is related to the fracture process zone with a certain area or volume (Bazant and Jirasek, 2002) which increases during deformation (Pijaudier-Cabot et al., 2004). It depends also on the choice of the weighting function. In turn, other researchers conclude that the characteristic length depends upon the boundary value problem (Ferrara and di Prisco, 2001).

The FE-analyses show that a classical non-local model (Eq. (2)) does not fully regularize a boundary value problem in elasto-plasticity (Brinkgreve, 1994; Bobinski and Tejchman, 2004). Therefore, a modified formula (according to Brinkgreve, 1994) was used to calculate the non-local softening parameter

$$\bar{\kappa}(\boldsymbol{x}) = \kappa(\boldsymbol{x}) + m\left(\frac{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\kappa(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}}{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\,\mathrm{d}\boldsymbol{\xi}} - \kappa(\boldsymbol{x})\right), \tag{4}$$

where m denotes a non-local parameter controlling the size of the localized plastic zone and the distribution of the plastic strain. For $m = 0$, a local approach is obtained and for $m = 1$, a classical non-local model is recovered. If the non-local parameter $m > 1$, the influence of non-locality increases and the localized plastic region reaches a finite mesh-independent size (Bobinski and Tejchman, 2004). To simplify the calculations, the non-local rates were replaced by their approximation $\Delta\kappa^{\mathrm{est}}$ calculated on the basis of the known total strain increment values:

$$\Delta\bar{\kappa}(\boldsymbol{x}) \approx \Delta\kappa(\boldsymbol{x}) + m\left(\frac{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\Delta\kappa^{\mathrm{est}}(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}}{\int_V \omega(||\boldsymbol{x} - \boldsymbol{\xi}||)\,\mathrm{d}\boldsymbol{\xi}} - \Delta\kappa^{\mathrm{est}}(\boldsymbol{x})\right) \tag{5}$$

with $\Delta\kappa^{\mathrm{est}}(\boldsymbol{x}) - \Delta\varepsilon_1(\boldsymbol{x})$ ($\Delta\varepsilon_1$ the increment of principal total strain). Eq. (5) enables to 'freeze' the non-local influence of the neighboring points and to determine the actual values of the softening parameters using