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Influence of wave generator profile on the pure kinematic error and centrodes of harmonic drive

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ABSTRACT

This paper addresses the kinematics of the harmonic drive (HD). The influence of the wave generator shape on the pure kinematic error of HD is herein discussed and quantitatively evaluated. In particular, this error has been plotted for the different working modes (i.e. considering different components as frame). Similarities between meshing of the wave generator profile with flexspline and cam-follower system have been observed. Moreover, the analytical expressions of relative motion centrodes have been deduced. These are believed to be useful for the design of conjugate profile teeth.

1. Introduction

The Harmonic Gear Drive (HD) was invented in 1955 by W.Musser [1, 2]. This device has been recognized for excellent weight to torque ratio, zero backlash and high reduction ratios. Due to its versatility, the applications span many engineering fields. Although its initial morphology is quite similar to the current one, many significant improvements have been implemented after its invention. For instance, the original trapezoidal tooth profile with a pressure angle, derived from an experimental analysis of the wave of deformation, has been changed in order to reduce friction losses.

Many investigations are available on different design features of the device and an exhaustive review is beyond the scope of this paper, but few works are worth mentioning.

The monograph of Ginsburg [3] was likely one of the first to propose different analyses and design models. For instance, the combined use of conventional drives and HDs was hinted as well as an analytic study on the shape of the flexspline (FS) forced deformation with a synthesis of the teeth engagement based on the trajectory of tooth centroid. In particular, for such shape a Résal curve [3, 4] has been proposed or other approximations such as an involute or conjugate circular arcs. In fact, the determination of the arc length in a Résal curve requires the evaluation of an integral and cannot be expressed in terms of elementary functions. Because of the direct influence of the wave generator (WG) shape, the cited monograph has been the starting point of this investigation.

The report of Tuttle [5] contains extensive theoretical and experimental analyses on the HD. These analyses include the evaluation of stiffness, kinematic error, dynamic response and energy losses.

Emelyanov [6] identified the sources of the kinematic error of an HD as a combined effect of primary errors from individual components such as errors of manufacture and assembly of the units. An equation has been proposed for computing the value and frequency of the kinematic error validated by means of experiments. Based on the assumption of an ellipse as the shape of

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the deformed FS, Hsia [7] proposed the purely kinematic model of the HD for the numerical computation of the kinematic error, and recognized the inherent non-homokineticity of the device irrespective of manufacture or assembly errors.

Kosse [8] estimated the difference of phase between the WG and the area of teeth meshing. High values of this difference will result in relevant deformation of the FS. In the analysis the WG shape has been assumed elliptical. Taghirad [9, 10] applied a Kalman filter estimation to cancel torque fluctuations due to angular speed irregularities. In particular, to improve performances, a torque controller was designed and implemented with satisfactory results.

Maiti and Roy [11] proposed as a WG shape a combination of circular and elliptical arcs. This in order to keep a certain amount of the meshing portion as close as possible to a circle. The paper includes also a study on tip interference of gear teeth with involute shape.

A more accurate theoretical and experimental analysis of the HD kinematic error is due to Ghorbel et al. [12]. It has been confirmed that the main frequency of the kinematic error is twice the one of the WG. Through different experiments the two major components of the kinematic error have been identified. The first one is the *pure* kinematic error and it is intrinsic with the working principle of the HD, such as the one analyzed by Hsia. The second is a flexibility induced component. Gandhi and Ghorbel [13, 14] developed closed loop control strategies to compensate the kinematic error also in presence of hysteresis effects.

Many improvements on the tooth profile have been suggested in a series of patents by Ishikawa [15–17], and Kayabasi and Erzincanli [18] developed a finite elements analysis to compute the stress in the FS and optimize the tooth shape to increase fatigue life.

Ishikawa in 1989 obtained a patent [15] for the shape of the tooth in which a rack approximation was used. Tooth shape was obtained by means of a similarity transformation of the tooth centroid trajectory.

In 1994 a patent by S.Ishikawa [16] contains a method of describing the WG with a Fourier series. The coefficients of this series were chosen to minimize the stress in the FS due to the change in curvature. Euler–Savary equation was applied to find the conjugate profiles for the HD.

Dong et al. [19, 20], after a rigorous analysis of the kinematics of the HD, obtained an analytic expression for the instantaneous transmission ratio as a function of number of teeth and shape of the WG.

Dong et al. [21] optimized the circular spline (CS) tooth profile to decrease dynamic loads during meshing and avoid FS fatigue fracture. The analysis revealed that about 18% of the teeth participate to the sharing of the contact force.

Zhao et al. investigated the hysteresis phenomenon derived from the harmonic reducer and established a theoretical model of the dynamical behaviour of space manipulator with HDs [22].

In this investigation the kinematics of the HD is addressed. The following contributions are believed to be novel:

- analysis of the influence of different WG profiles on the pure kinematic error;
- the treatment of the WG and the FS as a special case of a cam with a continuous flexible follower;
- computation of polodes parametric equations for the relative motion between the Cartesian reference system fixed with the CS and the one attached to a FS tooth.

The availability of centrodes may help further investigations on teeth meshing in HDs.

The paper is divided in three parts. The first one deals with the definition and computation of pure kinematic errors for different kinematic inversions of the HD. The computations are carried out assuming a 3-4-5 polynomial cam curve as WG shape.

The second part investigates the effect of the WG profile on the kinematic error and on meshing conditions. In the third part, the Cartesian parametric equations of centrodes are obtained and a numerical example is also presented.

2. Computation of pure kinematic error

The HD is made of three components (Fig. 1a): WG, FS and CS. The WG profile is initially assumed to have an elliptical shape and expressed by the equation $\rho(\theta)$ in polar coordinates. Z_{FS} and Z_{CS} are, respectively, the number of teeth of FS and CS. The WG can be viewed as a carrier that drives the FS, as in Fig. 1b [23].

Following traditional HD kinematic models (*e.g.* [3, 7]), in this analysis the FS is considered as a planar constant length line profile with bending elasticity. This assumption is supported by the results presented by Chen et al. [24] where the estimated average circumferential strain is of the order of 10^{-5}.

The rotation of the WG will cause the motion of all FS points. We define a central surface [3] as the one passing through the middle of the wall thickness of the FS. The section of this surface along a plane orthogonal to the rotation axis is a constant velocity curve $\mathcal{C}$. This curve could be the neutral fiber of the FS mid-layer. However, other parallel curves, such as the pitch curve, have only a constant difference in the radii of curvature [25]. If L is the total length of $\mathcal{C}$ and v the constant velocity of a generic point on $\mathcal{C}$ with WG fixed, then the following condition must hold

$$\int_0^{\frac{L}{v}} \omega dt = 2\pi \tag{1}$$

where ω is the angular velocity of the radius connecting the center of revolution with such point, as shown in Fig. 2a.