Mechanism and Machine Theory 55 (2012) 18–33

Contents lists available at SciVerse ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

A pseudo-rigid-body 2R model of flexural beam in compliant mechanisms

Yue-Qing Yu *, Zhong-Lei Feng, Qi-Ping Xu

College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100124, PR China

ARTICLE INFO

Article history:
Received 8 October 2011
Received in revised form 16 April 2012
Accepted 20 April 2012
Available online 18 May 2012

Keywords:
Pseudo-rigid-body model
2R PRBM
Flexural beam
Compliant mechanism

ABSTRACT

Based on the pseudo-rigid-body model (PRBM), a 2R PRBM that consists of three rigid links joined by two revolute joints and two torsion springs is proposed in this study. A method of parametric approximation to the deflection path and deflection angle of a flexural beam is developed for the 2R PRBM. A two-dimensional optimization for the characteristic radius factors and a linear regression for the spring stiffness coefficient are presented. Although the model parameters are dependent on the loading conditions, the 2R PRBM is useful in increasing the modeling accuracy of the 1R PRBM and reducing the computation time of the 3R PRBM. The advantage of the new model is also illustrated through a comparison of deformation energies among the various kinds of PRBM and the flexural beam. An application example of compliant mechanism is presented using the 2R PRBM. The 2R PRBM is significant to expand the applications of pseudo-rigid-body model in the analysis and design of compliant mechanisms, particularly in the further study on the dynamics of compliant mechanisms.

1. Introduction

The compliant mechanism is a new type of mechanism that uses the elastic deformation of components to accomplish the transmission of motion and force. Differing from the traditional rigid mechanism whose motion and function are accomplished by kinematic pairs, the main motion and function of a compliant mechanism are accomplished by the deformation of flexural components and then the motion, force and energy are also transmitted and transformed [1]. Compared with conventional rigid-body mechanisms, compliant mechanisms have many advantages in cost-reduction and performance-improvement such as part-count reduction, reduced assembly time, simplified manufacturing processes, reduced friction, wear, backlash and noise [2]. Compliant mechanisms bring revolutionary impacts and changes to the mechanical disciplines and engineering. As a new branch of modern mechanisms and machinery equipment, compliant mechanisms have a wide range of applications in the mechanical design, especially in the micro-electro-mechanical-system (MEMS) [3]. Therefore, an increasing attention has been attracted into the study of compliant mechanisms in recent years.

From the perspective of structural mechanics, Ananthasuresh [4,5] and Frecker [6] made extensive investigations and a number of achievements in the modeling, numerical analysis methods and software development of compliant mechanisms. Hetrik and Kota [7] introduced the topology optimization method to the design of compliant mechanisms. This method combined the size and shape optimization to perform the dimensional synthesis of mechanisms while simultaneously considering practical design specifications such as kinematic and stress constraints. An improved objective formulation based on maximizing the energy throughput of a linear static compliant mechanism was developed considering specific force and displacement operational requirements.

In the study of flexural mechanisms, Burns and Crossley [8] proposed a basic analysis method to simulate the flexural beam although the linear torsion spring was used and some assumptions were proposed. This pioneering work laid the foundation of the pseudo-rigid-body model (RPBM) [1] proposed later. The PRBM based on the structure and kinematics of rigid-body mechanism made an important progress in simplifying the analysis of compliant mechanisms. The main idea of PRBM is that in

---

* Corresponding author. Tel.: +86 10 67391702; fax: +86 10 67391617.
E-mail address: yqyu@bjut.edu.cn (Y.-Q. Yu).

0094-114X/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechmachtheory.2012.04.005

the motion of compliant mechanisms, the flexural beam is equivalent to a model comprised of two rigid links joined by one revolute joint and one torsion spring to represent the resistance of a beam's deflection. The PRBM can be used to simulate the motion of the flexural beam end by the rigid links joined at pivots and predict the force–deflection relationship by adding springs [9]. A parametric deflection approximation method for end-loaded, large-deflection beams in compliant mechanisms was applied to establish a simulation relationship between the PRBM and the flexural beam [10]. The path of the flexural beam end was accurately described by the end of the PRBM within 0.5% deflection error. An investigation of the spring stiffness in the PRBM was performed to simplify the complex force–deflection relationship in large deflection compliant mechanisms [11]. Then the idea of PRBM was also applied to the modeling of initially curved, large-deflection beams in compliant mechanisms [12]. Saxena and Kramer [13] modified the PRBM by introducing two linear springs to restrain the change of characteristic radius factor for different load modes. Lyon [14] decomposed a flexural beam into two segments, each of which is approximated by one PRBM. Saggere and Kota [15] proposed a finite element model in which a flexural beam can be modeled as several segments of PRBM.

From the perspective of manipulator kinematics, the joint connecting two links can be considered as a revolute pair. The model introduced above can be called 1R PRBM because the two links are connected with one joint, as shown in Fig. 1(a). The advantage of 1R model is its simplicity and so it is widely used in the analysis of compliant mechanisms. The workspace of 1R manipulator is the boundary of a circle, however, the deflection path of a flexural beam end is not an absolute circle. Therefore, the deflection path simulated by the 1R PRBM is not accurate in the whole range but approximate in certain ranges only. A qualification of deflection error is desired to decide the range of simulation. For example, when the qualification of deflection error is set to be 0.5%, the range of the deflection angle to be simulated is 0–124.4°. Moreover, although the path of flexural beam end can be simulated by 1R PRBM, the deflection angle of the beam end cannot be simulated because the 1R PRBM has only one degree of freedom (1-DOF). For the special case of modeling flexural beams with inflection points when the end moment load acts in the opposite direction as the end force, a 2-DOF PRBM [16] was proposed to improve the 1R PRBM.

Su [17] proposed a 3R PRBM that comprises four rigid links joined by three revolute joints and three torsion springs as shown in Fig. 1(b). It is well known that a 3R manipulator that has 3 DOF can accomplish accurately arbitrary location and pose of a planar link or beam, so the 3R PRBM can predict both the path and deflection angle of a flexural beam in a larger range compared with the 1R PRBM. The greatest strength of the 3R PRBM is its load independence no matter what kind of end-load is applied. The condition of load independence is satisfied by limiting the spring stiffness for two extreme loads, pure moment and vertical force. The benefit of load independence is critical for applications where loads vary significantly. However, the limitation of the 3R PRBM is that the inverse kinematic solution is harsh because three pseudo-rigid-body angles can be obtained only when the three parameters of beam end (two location parameters and one pose angle) must be fully given and so the computation time is very long. This may lead to the difficulty in the dynamics of compliant mechanisms.

On contrast, a 2R PRBM that has 2 DOF can provide a better solution for the angular deflection approximation than the 1R PRBM, and only two location parameters of the beam end are needed to solve and so the inverse kinematic solution of the 2R PRBM is much simpler than that of the 3R PRBM. This is significant to the dynamic analysis and design of compliant mechanisms.

In order to improve the modeling accuracy of the 1R PRBM and computation consumption of the 3R PRBM for the further study on the dynamics of compliant mechanisms, a 2R PRBM is proposed at first in this study. Three typical flexural cantilever beams with different end loads and the corresponding 2R PRBM are presented. The method of parametric approximation to the deflection path and deflection angle of the flexural beam is then introduced to optimize the characteristic radius factors and a linear regression for the spring stiffness coefficient of the 2R PRBM is presented. From the viewpoint of deformation energy, a comparison among the various kinds of PRBM and the flexural beam is made. An application example of compliant mechanism is also presented using the 2R PRBM. Some conclusions are made finally.

2. 2R PRBM with end force

2.1. Kinematical analysis

Fig. 2 shows a flexural cantilever beam with a force at free end and the corresponding 2R PRBM. The path of the beam end may be accurately modeled by three rigid links joined at two pivots. Torsion springs represent the resistance of the beam deflection.

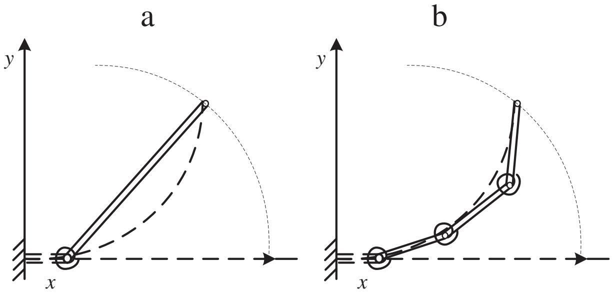

Fig. 1. (a) 1R PRBM. (b) 3R PRBM.