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Mobility and kinematic analysis of a parallel mechanism with both PPR and planar operation modes

Maurizio Ruggiu [a,*], Xianwen Kong [b]

[a] Department of Mechanical Engineering, University of Cagliari, Piazza d'Armi – 09123 Cagliari, Italy
[b] School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, UK, EH14 4AS

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ABSTRACT

This paper proposes a 1-R$\underline{P}$U-2-U$\underline{P}$U parallel manipulator with both *PPR* and planar operation modes. The mobility of the moving platform of the parallel manipulator in the two operational modes is analyzed via screw theory. The parallel manipulator has 3 degrees-of-freedom and allows the moving platform to move along a plane in the planar operation mode and to rotate about an axis that translates along a plane in the PPR operation mode. The transition between the two operational modes through transitional configurations is also discussed. The inverse and forward kinematic analysis and singularity analysis of the parallel manipulator are then dealt with by a vector approach. Finally, the constant orientation workspace and the reachable workspace are presented for the parallel manipulator in both operation modes.

1. Introduction

Numerous works on parallel manipulators (PMs) were dedicated to the synthesis and analysis of the limited mobility PMs: manipulators in which the moving platform has less than six degrees-of-freedom (DOF). The interest on limited mobility PMs is mainly due to the fact that several industrial tasks require 2–5 DOF. In most cases, limited mobility PMs can also be used as modules in hybrid mechanisms.

Up to now, most of the limited mobility PMs have a single operation mode, such as *planar, translational, spherical, Schönflies motion (3T1R)* (see [1,2] for example). Recently, there is an increasing interest toward PMs with multiple operation modes [3] (also *kinematotropic* [4,5] mechanisms, PMs that may change group of motion [6], PMs with motion bifurcation [7,8]), the moving platform of which can switch from operation mode to another. The change of the mobility of the moving platform of a PM usually occurs whenever the manipulator passes through a transitional configuration, which is usually a singular configuration called *constraint singularity* and *C-space singularity* by Zlatanov et al. [9,10].

Fanghella et al. [6] proposed a PM with both cylindrical and planar operation modes. Refaat et al. [7] proposed a new PM with motion bifurcation of the moving platform for machine tool application. Kong et al. [3] disclosed a family of PMs that can change working mode between 3-DOF spherical PMs and 3-DOF translational PMs. Li and Hervé synthesized a PM with bifurcation of Schönflies motion [8]. Recently Hernández et al. [11] showed that there are also poses in which, being no quantitative alteration of the freedom of the moving platform, there are changes in the type of the screw system spanned by the twist of the moving platform, *i.e.*, change of operational modes.

In this paper, the authors propose a PM with both PPR and planar operation modes and analyze the kinematic properties of the PM. Here a PPR motion mode refers to the operation mode in which the moving platform undergoes the same motion as the moving platform of a serial PPR manipulator. This follows the notation representing motion patterns in [2], which helps to avoid

* Corresponding author.
E-mail addresses: ruggiu@dimeca.unica.it (M. Ruggiu), X.Kong@hw.ac.uk (X. Kong).

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the ambiguity of representation of motion patters using the numbers of rotations and translations. Throughout this paper, *R*, *U* and P̲ denote a revolute joint, a universal joint and an actuated prismatic joint respectively. The manipulator has 3 DOF allowing the moving platform to move along a plane in the planar motion mode and to rotate about an axis that translates along a plane in the PPR operation mode.

In Section 2 the geometry of the PM under study is presented. In Section 3, the mobility of the moving platform will be analyzed using the screw theory. Sections 4, 5 and 6 are devoted to fully analyze the kinematic aspects of the PM. Finally, conclusions are drawn.

2. Geometry of the parallel manipulator

As pointed out in [3], the geometry of a PM with multiple operation modes can be clearly represented in its transition configuration, in which the PM can switch from one operation mode to another. Fig. 1 shows a transition configuration of the 1-RP̲U-2-UP̲U PM under study. The PM consists of a fixed base (A_i, $i=1,2,3$ as vertexes) and a moving platform (B_i, $i=1,2,3$ as vertexes) shaped as similar triangles connected by three legs. Leg A_1B_1 is a kinematic chain formed by *R*, P̲, *U* joint whilst the other two legs (A_2B_2 and A_3B_3) are formed by a sequence of *U*, P̲, *U* joints. In the *UPU* legs as well as in the *RPU* leg, the axes of two *R* joints connected by the *P* joint are parallel to each other. The direction of each P joint is perpendicular to the axes of its two adjacent R joints. In the transition configuration, all the axes of the R joints in the PM are parallel to the same plane. The link parameters of the moving platform are $\overline{B_2B_3}=2L$ and t (the distance from point B_1 to line B_2B_3). The link parameters of the base are $\overline{A_2A_3}=2R$ and h (the distance from point A_1 to line A_2A_3). In this paper, we assume that the link parameters of the PMs satisfy the following conditions: $t>h$ and $L\neq R$. The reasons why these assumptions are made can be found in the sections on kinematic analysis and singularity analysis.

In order to develop the kinematic analysis, two coordinate systems are defined.[1] A fixed reference system *O-xyz* and a moving reference system *P-uvw* on board of the moving platform. The origin *O* coincides with A_1 and the *x*-axis is perpendicular to $\overline{A_2A_3}$ while the origin *P* coincides with B_1 and *u*-axis is perpendicular to $\overline{B_2B_3}$.

As it will be proved later, the PM has the following two 3-DOF operation modes.[2] One may switch the PM from one operation mode to another via the transition configuration.

1. PPR operation mode: In each *UPU* leg the planes formed by the two *U* joints are parallel to each other (Fig. 2). In this case only a rotation ϕ about *y*-axis and translations along *y*- and *z*-axes are allowed.
2. Planar operation mode: The planes formed by all the *U* joints attached to the moving platform are all parallel (Fig. 3). The planes formed by all the *U* joints at the base coincide with plane $z=0$. In this case, only a rotation ψ about *x*-axis and translations along *y*- and *z*-axes are allowed.

3. Mobility analysis

In this section, the mobility of the moving platform will be obtained via screw theory following the procedure below.

1. Calculate the twists: ${}^j\$_i$ for $i=1,..,n$ and $j=1,\ldots,m$. Here *m* is the number of legs of the mechanism, and *n* is the number of DOF in the leg *j*.

$$ {}^j\$_i=\begin{bmatrix}\mathbf{s}_i\\ \mathbf{r}_i\times\mathbf{s}_i+h_i\mathbf{s}_i\end{bmatrix}\text{if } h_i \text{ is finite}, {}^j\$_i=\begin{bmatrix}0\\ \mathbf{s}_i\end{bmatrix}\text{if } h_i\rightarrow\infty. \tag{1}$$

where $\mathbf{s}_i$ is the unit vector of the screw, $\mathbf{r}_i$ is a vector from the origin of the leg's reference system to any point of the screw, h_i is the screw's pitch.

2. Construct the matrix of the leg twists ${}^j\mathbf{H}$ such that:

$$ {}^j\mathbf{H}=\begin{bmatrix}{}^j\$_1^T\\ \vdots\\ {}^j\$_i^T\\ \vdots\\ {}^j\$_n^T\end{bmatrix} \tag{2}$$

3. Calculate the wrenches (reciprocal screws) ${}^j\$_k^r$, ($k=1,\ldots,v$), as the basis of the null space of ${}^j\mathbf{H}$. According to the rank-nullity theorem, the dimension of the null space $nullity({}^j\mathbf{H})\equiv v$ is given as $v=6-rank({}^j\mathbf{H})$. Since a vector ${}^j\$_k^r$ lies in the null space of ${}^j\mathbf{H}$ if and only if it is orthogonal to every vector in the row space of ${}^j\mathbf{H}$, the null space calculation is equivalent to the reciprocity between twist-and-wrench in the leg. Therefore, a constraint wrench ${}^j\$_k^r$ denotes a wrench *k* which produces no power for each twist *i* of the leg *j*.

[1] Rotations and displacements are taken to be positive according to the positive directions of the axes/directions about/along which the motions occur.

[2] As one of the reviewers' pointed out, one may also prove that the proposed PM has both the PPR and planar operation modes using the intersection of 4- and 5- dimensional manifolds of rigid-body displacements.