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Numerical plane stress elastic–perfectly plastic crack analysis under Tresca yield condition with comparison to Dugdale plastic strip model

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Abstract

A number of plane stress numerical analyses of the mode I elastoplastic fracture mechanics problem have been performed in the past using the Huber–Mises yield criterion. This study employs instead the Tresca yield condition using an incremental theory of plasticity for a stationary crack. A commercial finite element program is used to solve the opening mode of fracture problem (mode I) for a square plate containing a central crack under generalized plane stress loading conditions. A biaxial uniform tensile traction is applied to the edges of a thin plate composed of a linear elastic non-work hardening material under small strain assumptions. The finite element results are compared with the analytical predictions of the Dugdale plastic strip model for a crack in an infinite plate subject to a biaxial uniform load at infinity.

Keywords: Mode I; Crack; Linear elastic; Perfectly plastic; Tresca yield condition; Dugdale model

1. Introduction

Numerical methods of solution have been applied previously to plane stress elastoplastic fracture mechanics problems satisfying the Huber–Mises yield condition (maximum distortion energy criterion) for stationary crack problems (Tuba, 1966; Kudryavtsev et al., 1970; Hilton and Hutchinson, 1971; Narasimhan and Rosakis, 1988; Dong and Pan, 1990; Sham and Hancock, 1999). However, only a plane strain finite element analysis has been published to date using the Tresca yield condition (maximum shear stress criterion) for a geometry similar to the mode I problem, where a notched plate rather than a line crack is considered (Owen et al., 1973).

While no exact analytical elastoplastic solution has been found for a plane mode I crack problem, one approximation for which an analytical solution exists is the plane stress plastic strip model (Dugdale, 1960). This model assumes the existence of two infinitesimally narrow strips of plastic material positioned

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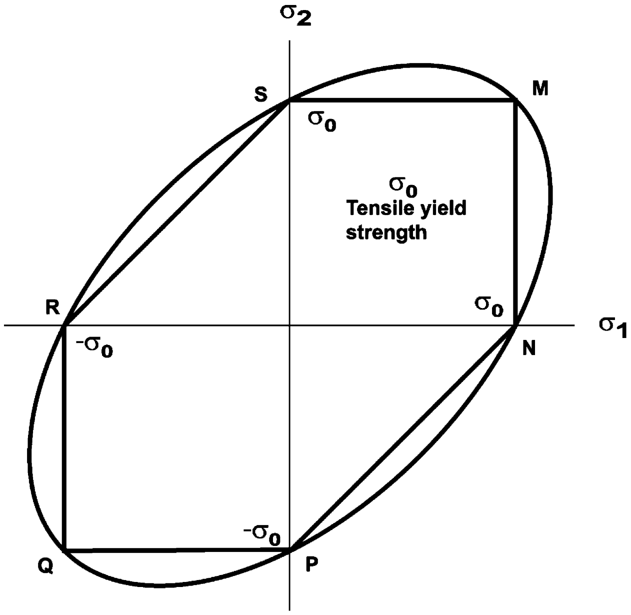

Fig. 1. Huber–Mises (elliptical) and Tresca (hexagonal) yield loci.

ahead of individual tips of a finite length crack in an infinite medium. Along these plastic strips the stresses satisfy the Tresca yield condition (point M of the yield surface shown in Fig. 1). These plastic strips are treated mathematically as applied tensile tractions along portions of an extended crack in an otherwise linear elastic crack problem. Dugdale also provided supporting experimental evidence of the formation of narrow plastic strips ahead of crack tips in thin plates of mild steel. Other researchers have also reported their formation (Hahn and Rosenfield, 1965; Rosenfield et al., 1966) and in Hahn and Rosenfield (1968) it was suggested that elongated plastic zones may be indicative of materials exhibiting perfectly plastic post yield behavior under plane stress loading conditions. The analysis provided here lends support to this assertion.

2. Analysis

Here a preliminary study of a mode I plane stress crack problem employing the Tresca yield condition and an incremental theory of plasticity is presented for a square plate of elastic–perfectly plastic material containing a stationary crack. These numerical results are compared to the predictions of the Dugdale plastic strip model.

A square plate 16 in. on each side with a thickness of 0.01 in. is examined. Only one quarter of the plane surface is modeled due to symmetry. See Fig. 2 for a schematic drawing of the imposed boundary conditions. The number of square quad 4 elements chosen for each side of the quarter plate is 101, thus making the total number of finite elements 10,201. One half of the internal crack length a is composed of 11 elements for a total length of 0.871 in., as shown in Fig. 3. The material parameters chosen for this analysis are: Young's modulus $E = 30 \times 10^6$ psi, Poisson's ratio $\nu = 0.3$, and a tensile yield strength $\sigma_0 = 100{,}000$ psi. The tensile traction applied to all four sides of the square plate is $\sigma_\infty = 70{,}000$ psi. The commercial finite element program employed in this study was Nastran version 70.5 implemented through the graphical interface capabilities of Patran version 8.0 of the MacNeal–Schwendler Corporation.

In Burdekin and Stone (1966), the Westergaard function Z_{I} for the Dugdale plastic strip model for a crack of physical length $2a$ centered along the x-axis is given as

$$Z_{\mathrm{I}} = \frac{\sigma_0}{\pi}\cot^{-1}\left(\frac{a}{z}\sqrt{\frac{z^2 - c^2}{c^2 - a^2}}\right), \tag{1}$$