Mechanism and Machine Theory 99 (2016) 189–206

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Avoiding the singularities of 3-R$\underline{\text{P}}$R parallel mechanisms via dimensional synthesis and self-reconfigurability

Amirhossein Karimi [a], Mehdi Tale Masouleh [a,*], Philippe Cardou [b]

[a] Human and Robot Interaction Laboratory, Faculty of New Sciences and Technologies, University of Tehran, Iran
[b] Department of Mechanical Engineering, Laval University, Quebec City, QCG1V 0A6, Canada

ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form 7 January 2016
Accepted 11 January 2016
Available online 3 February 2016

Keywords:
Parallel mechanism
Dimensional synthesis
Self-reconfigurable mechanisms
Convex programming
Singularity-free workspace

ABSTRACT

This paper deals with the avoidance of singularities of 3-R$\underline{\text{P}}$R parallel mechanisms, for a given workspace. Two approaches are proposed: (1) the dimensional synthesis of such robots for a prescribed singularity-free workspace, and (2) their reconfiguration via actuation redundancy. The proposed approach for the dimensional synthesis of 3-R$\underline{\text{P}}$R parallel mechanisms considers the constraints required for obtaining the optimal mechanism geometry in order to contain a desired workspace while avoiding singularities. A relaxation method based on the McCormick relaxation is presented to convexify the associated optimization problem. To prevent the relaxation from becoming too loose, a branch-and-prune algorithm is proposed, which converges to the globally optimal solution. Upon a different approach, a 3-R$\underline{\text{P}}$R PM with self-reconfiguration capability is introduced. This redundant manipulator can change its geometry to obtain singularity-free paths between prescribed points inside its workspace. The proposed trajectory planning strategy works in 2 phases: the first where the mechanism acts as a 2-$\underline{\text{P}}$RPR PM, and the second where it behaves as a 3-R$\underline{\text{P}}$R PM. Finally, several case studies are presented to demonstrate the performance of the proposed algorithms. The running times of the proposed algorithms are remarkably low compared to those of other methods proposed in the literature.

1. Introduction

Parallel robots play an important role in industry nowadays, as witnessed by an increasing growth in a variety of their applications. High load-carrying capacity, better accuracy and higher velocities and accelerations are of their advantages by comparison with serial manipulators, which make them widely used in automotive, aircraft, and packaging industries. However, the analyses of their kinematics, workspace, and singularities are of considerable complexity versus serial robots, as some of the problems in this field have been remained unsolved. Confrontation with singularities of parallel robots can be expressed in terms of three different strategies, namely: 1) obtaining largest singularity-free zones within the workspace of the mechanisms to ensure the safety of these regions from singularity, 2) synthesis considerations for designing mechanisms with fewer singularities, 3) to consider the singularity in the path generation procedure to obtain singularity-free paths. There are many papers in the literature contributed to the first strategy [1]. This paper aims at proposing analytical methods for other two strategies in order to resolve problems caused by the singularities in the

* Corresponding author.
E-mail addresses: ah.karimi@ut.ac.ir (A. Karimi), m.t.masouleh@ut.ac.ir (M. Masouleh), pcardou@gmc.ulaval.ca (P. Cardou).

http://dx.doi.org/10.1016/j.mechmachtheory.2016.01.006

workspace of the mechanisms, including failure to implement control algorithms, collapse of mechanism configuration, and damage to the mechanical components of the robot. At the outset, considering the desired singularity-free workspace and the specified geometric constraints, it is aimed at providing an efficient method for the synthesis of 3-R$\underline{P}$R parallel mechanism (PM). It is obvious that a synthesized mechanism cannot fulfill all the requirements. For instance, it may be required for the mechanism to track a singularity-free course which does not lay within its pre-defined singularity-free workspace. In this case, self-reconfigurability is desired for the mechanism in order to resynthesize and change its configuration based on the required singularity-free path. Therefore, in the second strategy, a 3-Degree-Of-freedom (DOF) mechanism with self-reconfigurability will be introduced, which is able to track any desired path within its workspace without crossing the singularities. The axial novelty of the present research is combining convex optimization approaches with the kinematics of parallel robots which has the positives of not trapping in local optimums as well as low computational burden. The latter is of high importance in the second strategy since it can pave the way for the online application of self-reconfigurability.

The synthesis of parallel mechanisms is much more complex than that of serial robots, due to many reasons such as the increased number of parameters required to determine the geometry of the mechanism and complicated analysis of workspace and singularity configurations. The studies conducted in the synthesis of parallel mechanisms are mostly based on heuristic methods such as genetic algorithm [2,3], and interval analysis [4] or using a sequential procedure in which two arms are designed first to satisfy the workspace requirements, then the third arm is designed to provide a singularity-free workspace [5]. It should be noted that in the latter, a feasible subset for the problem of the dimensional synthesis of a 3-R$\underline{P}$R parallel mechanism is obtained in order to contain a prescribed workspace while it is singularity-free. Moreover, in Refs. [6,7] a geometric method is introduced for the synthesis of a 3-R$\underline{P}$R PM, in which the base and mobile platform triangles are geometrically similar to each other.

There are various indices according to which a parallel mechanism can be designed. It is indispensable for a mechanism which performs any special task, to have a prescribed workspace. Therefore, the workspace area and shape are of the foremost indices which can be taken into account for the purpose of dimensional synthesis. Another important criterion for the aim of synthesis is the presence of singularities. Furthermore, in some applications two or more separated parts may be prescribed as the desired workspace. This paper deals with these two indices for the purpose of synthesizing of 3-R$\underline{P}$R parallel mechanisms.

It is more than a decade that the study of the mechanisms with reconfigurability and the ability to change the motion pattern has attracted much attention in the robotics community [8–16]. Since the conventional mechanisms may involve many practical problems in multi-task applications, a variety of new mechanisms have been introduced, of which we can enumerate two well-known types as kinematotropic linkages [17] and metamorphic mechanisms [18]. These two types can change their operation modes during their motions in the environment. Alteration of the operation mode in kinematotropic linkages, such as those proposed in Refs. [19–21], is implemented by passing through constraint singularities [22,23]. This is conducted in metamorphic mechanisms based on changing the topology and thus the mobility of the mechanism via joining links together and locking actuators during operation. One of the latter self-reconfigurable mechanisms is a 3-($\underline{P}$)$\underline{R}$RR PM, which has a prismatic redundancy in one of its limbs [24]. Moreover, in Ref. [25] a new geometric technique based on the use of CAD methodologies for determination of the total workspace of all planar parallel robotic manipulators has been presented as well as a graphical methodology for determining the singularity-free regions within the workspace. Besides, it is illustrated via a purely geometric method, how a designer can generate the singularity-free trajectories by switching between assembly modes. In Ref. [26], an algorithm for computing singularity-free paths on non-redundant closed-chain manipulators such as 3-RPR and 3-RRR is provided. In Ref. [27], a novel family of singularity-free kinematically redundant planar parallel mechanisms that have unlimited rotational capabilities are introduced which are akin to conventional Three-Degree-Of-freedom planar parallel mechanisms. By introducing a novel kinematically redundant arrangement, 4-DOF parallel mechanisms are obtained that can completely alleviate singularities and provide unlimited rotational capabilities [27].

The remainder of the paper is organized as follows. First, in Section 2, the problem formulation for the synthesis of a 3-R$\underline{P}$R PM is described and the optimization problem is expressed by introducing the constraints required for the optimal geometry to contain the prescribed workspace while avoiding singularities. Then, a relaxation method based on the McCormick relaxation is presented to convexify the non-convex constraints. A branch-and-prune algorithm is then proposed, which converges to the global optimal solution. Thereafter, several case studies are examined in order to clarify the proposed algorithm. In Section 3, a novel 3-R$\underline{P}$R PM with self-reconfigurability is described. Moreover, the problem formulation is analyzed in order to obtain a geometry for the 3-R$\underline{P}$R PM, for which the desired line segment lies within the singularity-free workspace. Then, a case study is explored based on the proposed algorithm. Finally, the paper concludes by providing some remarks and describing related ongoing work.

2. Synthesis problem

As discussed previously, one approach to avoid the singularities of parallel robots is to synthesize them with fewer singularities from the outset. In this section, we formulate a dimensional synthesis problem for a 3-R$\underline{P}$R PM with this approach in mind, and we use a convex optimization method to solve the problem.