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Forced response of low-frequency pendulum mechanism

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ABSTRACT

A strongly nonlinear pendulum mechanism is considered in which the restoring force is approximately a cubic function of the displacement variable. Its free oscillation frequency is approximately proportional to the amplitude of oscillation and distinctly lower than that of a simple pendulum. The mechanism has therefore been named infra-pendulum. The forced undamped oscillation response of the mechanism to non-harmonic periodic loading is studied under the assumption of small displacements. The loading function is derived from the free oscillation response whose time course follows a Jacobi elliptic function. It is chosen such that exact analytical solutions are obtained for the steady-state response and the amplitude–frequency relation. The equation describing the amplitude–frequency relation is a cubic polynomial equation. Its solutions are presented. The general approach of using non-harmonic periodic loading functions is transferable to other types of nonlinear oscillators.

1. Introduction

The development of the mechanism studied here was inspired by the search for low-frequency pendulum mechanisms. Such mechanisms have applications in engineering and science [1]. They can possibly be used as passive mass dampers for large engineering structures, which have low natural frequencies of vibration. Tuning a conventional pendulum to a low frequency requires a correspondingly large rod length, which not always can be accommodated. Furthermore, conventional passive mass dampers become inefficient if they are not precisely tuned to the frequency of the vibration to be damped. Nonlinear mechanisms, as the one studied here, may offer the advantage of reduced tuning accuracy demand.

The mechanism is shown in Fig. 1. It consists of a massless rod or cord of length H suspended from a hinge at its upper end, a massless rigid beam hinged to the lower end of the rod, a frictionless vertically sliding lateral support at a certain point of the beam, and two identical point masses $m_1 = m_2 = M/2$. The masses are attached to the beam on opposite sides of, and at the same distance L, from the lower hinge (i.e., the hinge at the lower end of the rod). The distance along the beam between the lower hinge and the lateral support is called l. Note that l can also be equal to or greater than L. The gravitational acceleration is denoted by g. Horizontal, in the foregoing sentences, means perpendicular to the direction of gravity. Only displacements in the drawing plane are allowed.

The mechanism has one degree of freedom. Fig. 1 shows the static equilibrium position, in which the rod is oriented vertically and the beam horizontally. The equilibrium is stable. Fig. 2 shows the mechanism in a displaced position, where the beam has moved vertically but not horizontally at the sliding support.

Let's assume that there is no external force applied, that is, $F(t) = 0$. After inducing a displacement, the beam oscillates around its equilibrium position in a combined rotational and translational motion, whereas the rod, due to the kinematic constraints, swings to only one side of its equilibrium position. The masses move in mainly vertical and opposite directions (as long as the

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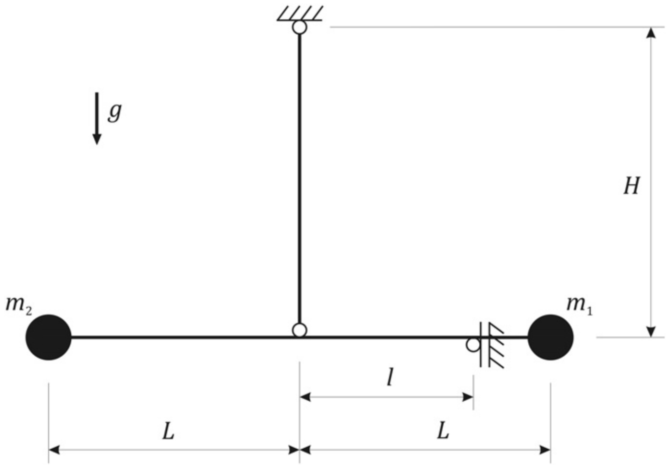

Fig. 1. Infra–pendulum in static equilibrium position.

displacement is small). The swinging motion of the rod goes along with a periodic lifting of the center of gravity, which coincides with the position of the lower hinge. Both these motions are small in relation to the motions of the individual masses. Hence the restoring forces, compared to a simple pendulum, are likewise small in relation to the inertia forces and, thus, the free oscillation frequency is comparatively low. Therefore, the mechanism has been named infra-pendulum [2].

The free undamped oscillation response of the mechanism was studied in a previous publication [2]. The most important results are summarized in the next section, which also includes an improved derivation of the free oscillation equation of motion. On this basis, the forced undamped oscillation response to a periodic loading is studied in the main part of this work. Displacements are assumed to be small.

2. Free undamped oscillation response

When the external force $F(t)$ indicated in Fig. 2 is zero, a free undamped oscillation response of the infra-pendulum can occur. The equation of motion and its solution can be developed in terms of the beam rotation, α, or, as preferred here, the displacement variable $v = l\sin\alpha$, which is the rise of the beam at its lateral support minus the rise of the lower hinge, y (Fig. 2). Both these

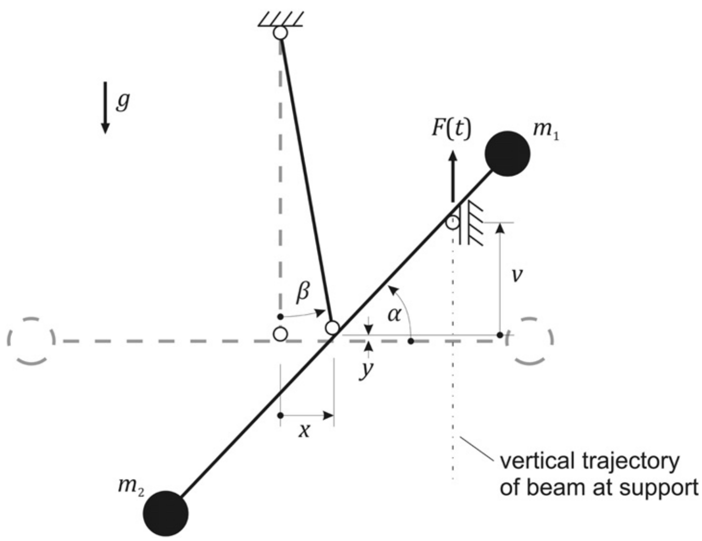

Fig. 2. Infra–pendulum in displaced position.