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Nonlinear vibration control of a cantilever beam by a nonlinear energy sink

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ABSTRACT

This paper investigates the effect of an attached/coupled nonlinear energy sink (NES[1]) on energy suppression of a cantilever beam under shock excitation. Grounded and ungrounded configurations of NES are studied and the NES performance is optimized through variation of different parameters. The realization of nonlinear vibration control through one-way irreversible nonlinear energy pumping and optimizing the system parameters result in acquiring up to 89% dissipation of the ungrounded system energy imposed by shock excitation. In addition, the bifurcations and topological structure of nonlinear normal modes (NNMs[2]) of both systems are studied in order to find necessary conditions of targeted energy transfer (TET[3]) realization in each configuration. In fact, the role of NNMs in identifying dynamics of energy pumping in especially continuous systems is studied for the first time.

1. Introduction

A great attention has been recently paid to employing NES as an essential nonlinear energy absorber rather than linear absorbers or weakly nonlinear absorbers.

Applications of the NES in 2-DOF or multi-DOF systems of weakly coupled linear and essentially nonlinear damped oscillators with different parameters and conditions were studied widely for nonlinear energy pumping and a variety of system response regimes in [1–5]. 2DOF systems comprised of a linear oscillator and an attached NES with pure cubic nonlinearity were considered in [6–13], as well. Moreover, in studies [14–15], application of the NES in attenuating the self-excited oscillations of a van der Pol oscillator, and also different response regimes were examined. On the other hand, dissipated energy by an attached NES to a simply supported beam was examined in [16].

The importance of the topological structure of NNMs of a non-dissipative and unforced system in TET of a dissipative system was studied for a multi-DOF system of linear oscillators coupled to a NES in [17]. Indeed, it was proven that dynamics of energy pumping mainly relied on the structure of these modes for discrete systems.

In this work, for the first time a linear continuous system (a finite length beam) with two different NES configurations (grounded and ungrounded) will be considered for optimizing the amount of energy dissipation. Then, the conditions of TET realization in each system will be introduced through evaluation of the NNMs structures. In fact, by comparing the results obtained from dissipated and instantaneous energies of each system with the structure of corresponding NNMs, the important role of NNMs in irreversible energy pumping will be proven for continuous systems.

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[1] Nonlinear energy sink.
[2] Nonlinear normal modes.
[3] Targeted energy transfer.

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2. Model analysis

Consider a cantilever beam which is attached to an NES and is under shock excitation. Nonlinear attachment is examined in two different configurations as shown in Fig. 1: grounded and ungrounded. In the grounded configuration, the NES is weakly coupled to the beam.

Applying Euler–Bernoulli beam theory, the equations of motion for two configurations would be in the following forms:

Grounded configuration, NES#1,

$$\begin{aligned}&EIy_{xxxx}(x,t)+My_{tt}(x,t)+\varepsilon\beta y_t(x,t)+\varepsilon k_1[y(d,t)-w(t)]\delta(x-d)=F(t)\delta(x-s)\\&\varepsilon M\ddot{w}(t)+\varepsilon k_1[w(t)-y(d,t)]+kw^3(t)+\varepsilon C\dot{w}(t)=0\end{aligned} \tag{1}$$

Ungrounded configuration, NES#2,

$$\begin{aligned}&EIy_{xxxx}(x,t)+My_{tt}(x,t)+\varepsilon\beta y_t(x,t)+\left\{\varepsilon k[y(d,t)-w(t)]^3+\varepsilon C[y_t(d,t)-\dot{w}(t)]\right\}\delta(x-d)=F(t)\delta(x-s)\\&\varepsilon M\ddot{w}(t)+\varepsilon k[w(t)-y(d,t)]^3+\varepsilon C[\dot{w}(t)-y_t(d,t)]=0\end{aligned} \tag{2}$$

where dot denotes the differentiation with respect to t, and subscript x or t shows partial differentiation; E and EI are the Young's modulus and bending stiffness of the beam, respectively; M is the beam mass; k_1 and k are the coupling spring stiffness and the spring stiffness of the NES; β and C are the damping coefficients; $\delta(x-d)$ is the Dirac function; s and d are the place of applying the external load and attaching the NES, respectively; $\varepsilon \ll 1$. It should be mentioned that since the comparison between NES#1 and NES#3, shown in Fig. 1c, is not possible (because the NES is free of load and does not play any role in this system), the comparison will be made between NES#1 and NES#2.

The nth vibration mode Φ_n and frequency ω_n of a cantilever beam are,

$$\Phi_n(x)=A_n\{[\sin(\beta_n L)-\sinh(\beta_n L)][(\sin(\beta_n x)-\sinh(\beta_n x))]+[\cos(\beta_n L)+\cosh(\beta_n L)][(\cos(\beta_n x)-\cosh(\beta_n x))]\}$$

$$\omega_n=\beta_n^2\sqrt{\frac{EI}{M}}$$

$$A_n=\left\{\int_{x=0}^{L}M\left[\frac{\Phi_n(x)}{A_n}\right]^2dx\right\}^{\frac{-1}{2}} \tag{3}$$

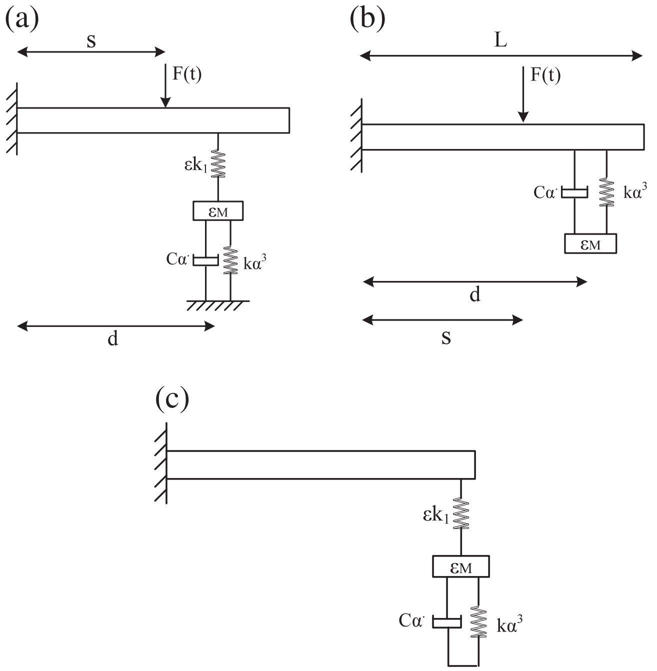

Fig. 1. NES configurations: (a) configuration of NES#1, (b) configuration of NES#2, (c) configuration of NES#3.