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Yield criterion for porous media with spherical voids

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Abstract

The yield criterion of a porous material satisfying Gurson criterion conditions is studied here. We use the twofold limit analysis approach applied to a representative volume element of the porous media and recent optimization codes. Both upper and lower bounds are very close, and they give quasi-exact solutions. As a result, the Gurson approach is slightly improved and, *for the first time*, validated by a rigorous, full 3D static approach.

Keywords: Limit analysis; Conic optimization; Finite elements; Porous media; Yield criterion

1. Introduction

In ductile failure of porous materials, the Gurson criterion (1977) is the most widely accepted because it is based on a homogeneization method and on the kinematic approach of limit analysis. The plastic domain is approached from the outside by a rigorous semi-analytical approach. Gurson's model concerns porous isotropic materials with macroscopic strain imposed on the boundary. The original criterion for a perfectly plastic rigid matrix around a spherical cavity is expressed as follows:

$$\frac{\Sigma_{\mathrm{eqv}}^2}{3k^2} + 2f\cosh\left(\frac{\sqrt{3}\Sigma_{\mathrm{m}}}{2k}\right) = 1 + f^2 \tag{1}$$

where f is the porosity rate of the studied material and k the flow stress in shear or the cohesion. Σ_{eqv} is the macroscopic equivalent stress and Σ_{m} the macroscopic mean stress. By definition, this model does not take into account interactions between cavities or coalescence.

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To obtain the plastic domain in the macroscopic stress space of a material checking Gurson criterion conditions, several tools are developed. First, homogeneization theory makes it possible to model the real material by a representative volume element (RVE) with all information required for the description of the real structure.

The homogeneization method links macroscopic stresses and strain velocities Σ_{ij} and E_{ij} with microscopic stresses and strain velocities σ_{ij} and v_{ij} by the average relations

$$\Sigma_{ij} = \frac{1}{V}\int_V \sigma_{ij}\,\mathrm{d}V \quad E_{ij} = \frac{1}{V}\int_V v_{ij}\,\mathrm{d}V \tag{2}$$

This RVE can be subjected to either the macroscopic strain rate imposed at the boundary of RVE ($u_i = E_{ij}x_j$, uniform strain rate) or the macroscopic stress ($\sigma_{ij}n_j = \Sigma_{ij}n_j$, uniform stress) leading to two different mechanical problems. The real criterion is located between the solutions of these problems.

Here, we analyse the case of uniform strain boundary conditions for comparison with the original Gurson criterion.

Then, we use the static and kinematic limit analysis approaches. They lead to a nonlinear optimization problem (because of the plasticity criterion) solved independently by XA or MOSEK, both optimization codes use interior point algorithms.

XA is a linear programming (LP) code; that is why the linearization is done beforehand using the Ben-Tal and Nemirosvski method (BTN) (Ben-Tal and Nemirovski, 2001) applied in our work in Pastor et al. (2004). The drawback of this linearization is the amount of memory required for large-scale problems. MOSEK is a nonlinear (conic) optimization code; it solves the problem directly and thus deals with thinner RVE mesh, specially in the 3D case.

In this paper, the admissible loading domain of the Gurson problem is sought using a three-dimensional hollow sphere as the RVE. In Section 2, *a plane mesh* is used to model the RVE subject to an axisymmetric loading. This first model does not provide a sufficiently accurate yield criterion. Therefore, in Section 3 *a three-dimensional mesh* (tetrahedron elements) is used. This model gives a quasi-exact yield criterion, improving the Gurson criterion slightly. It does not confirm the corner found in Francescato et al. (2004) and Pastor et al. (2004) on the horizontal mean stress axis in the case of a porous material with cylindrical cavities. More precisely, the kinematic 3D bound is very close to the kinematic axisymmetric bound, which improves the Gurson approach slightly. This shows that the *kinematic axisymmetric approach* is excellent. The static 3D approach improves the static axisymmetric approach slightly and comes very close to the kinematic approach. Then the Gurson criterion, *for the first time*, is validated both with our kinematic and static approach. Moreover, the kinematic approach is reinforced as a prevision tool of macroscopic isotropic criterion.

2. Axisymmetric method

2.1. Description of the problem

The 3D problem is solved here using a plane mesh. To model the RVE, a quarter of a ring is meshed with triangular elements in the reference frame ($OR\theta Z$). This frame rotates around the Z axis and a half-hollow sphere is thus obtained (Fig. 1). Indeed, the loading must also be axisymmetric.

The total power P^{tot} can be written as follows, with the actual loading parameters:

$$P^{\mathrm{tot}}/V^{\mathrm{tot}} = \Sigma_{\mathrm{m}}E_{\mathrm{m}} + \Sigma_{\mathrm{ps}}E_{\mathrm{ps}} + \Sigma_{\mathrm{gps}}E_{\mathrm{gps}} \tag{3}$$

where the macroscopic stresses and the macroscopic strain velocities are defined by