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Analysis of the center of rotation in primitive flexures: Uniform cantilever beams with constant curvature

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ABSTRACT

Precision of rotation is a fundamental characteristic of primitive flexures, especially when compliant mechanisms are designed for micro-scale applications. In case of single leaf flexures, distributed compliance allows flexible beams to undergo large deflections. Nevertheless, the center of rotation changes its position during such deflections. In this paper, large deflection analysis is performed on cantilever beams with uniform cross section and constant curvature, subjected to end-moment loads. Analytical expressions to determine the position of the center of rotation of the flexure are derived. The center of rotation refers to the displacement of the free-end section of the beam, occurring when the flexure, due to the external load application, moves from its neutral configuration to the deformed one. Several examples are considered and the analytical solutions are compared to the results obtained by finite element analysis. A final example focuses on the determination of the pseudo-rigid body model.

1. Introduction

In the last decades, compliant mechanisms have been developed and adopted in various fields, especially at the micro-scale. Many applications can be found, for example, in the areas of micro-manipulation [1–4], micro-positioning [5–9], precise machining/manufacturing [10,11], and, broadly, MEMS [12–15]. With the increasing demand of compliant mechanisms as key tools in such fields, the requirements for motion resolution and positioning accuracy, but also the precision of the analytical models, represent very important issues [16,17].

Compliant mechanisms rely on the elastic deformations of their flexible parts, or flexures, to transmit force or motion. In their simplest form, *primitive* flexures are essentially notch-type joints [18–21] and flexible beams [22–25]. Other types of flexures, often called *complex* flexures, have been developed combining more flexible elements or involving contact systems [26–29]. Complex flexures are usually designed to improve the performance of compliant systems in terms of stress concentration, off-axis stiffness to axial stiffness ratio, range of motion, and precision of rotation [30]. Generally, the last two characteristics work against each other, and developing advanced flexures able to achieve wide range of motion and high precision of rotation has been the goal of recent investigations. The butterfly flexure pivot proposed by Henein et al. [31], and the anti-symmetric double leaf-type isosceles-trapezoidal flexure joint (ADLIF) introduced by Pei and Yu [32], are notable examples.

The relation between imprecise rotation and wide range of motion stems from the fact that, during the load application and the subsequent flexure deflection, the center of rotation does not remain fixed with respect to the connected links. This condition, referred to as axis shift, center shift, parasitic center shift or parasitic motion, was analyzed in several studies focused on complex

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flexures, such as butterfly pivots [33,34], leaf-type pivots [35–37], and lumped or distributed compliant parallel-guiding mechanisms [38,39].

In fact, large range of motion and high precision of rotation cannot be easily achieved using primitive flexures. More specifically, small-length notch-type joints are characterized by high-precision of rotation and high off-axis stiffness, but also by limited angular displacement and high stress concentration. On the contrary, at the cost of lower precision of rotation, distributed compliance allows flexible beams to achieve a wide range of motion undergoing large deflections [40]. Hence, large deflection and precision of rotation analysis represent two fundamental aspects in flexible beams designed as primitive flexures.

As key elements of mechanical systems, beams undergoing large deflections have been studied for decades. The firsts investigations were focused on straight or curved cantilever beams subjected to concentrated or distributed loads [41–44]. Recently, various load conditions, such as follower forces or combined loads, and solution techniques, such as integral approaches and direct nonlinear solution [45–50], have been considered.

With respect to compliant mechanisms, large deflections analysis is a prerequisite to the determination of the pseudo-rigid body model (PRBM), a useful tool for both analysis and synthesis problems [51,52]. In this model, the flexible element is replaced by a pin joint connecting two rigid links, whose position is determined in order to better approximate the path followed by the free-end section of the beam. The PRBM can simplify a lot the analysis and the synthesis of compliant mechanisms, since it allows rigid links mechanism theory to be applied. However, as approximated model, it could show errors not negligible in high precision applications. To reduce such errors, other researchers proposed PRBMs with two or three revolute joints. Therefore, two- or three-dimensional optimization techniques were adopted to find the characteristic radius factors and the springs stiffness coefficients best approximating the beam tip deflection path, the deflection angle, and the beam stiffness [53–55].

Although many studies focused on large deflections examining various load conditions and solution techniques, the position of the center of rotation received little attention. Furthermore, the center shift is usually not considered in PRBMs [56]. On the other hand, from the flexures viewpoint, some investigations considered parasitic motions in complex flexures rather than in primitive flexures as single leaf segment.

In a recent investigation, as possible strategy to set-up PRBMs, Belfiore proposed the center of rotation of the flexible element tip as position of the revolute joint. The Principle of Virtual Work was then applied to find the position and the orientation of the free-end section of the cantilever beam [57].

In this scenario, an analytical tool for determining the center of rotation positions of flexible beams could contribute to (i) the determination of accurate PRBMs, and (ii) the evaluation and the improvement of the flexure performance in terms of precision of rotation. For these reasons, in this paper, large deflection analysis is performed on cantilever beams with constant curvature, uniform cross section, and subjected to a moment applied to the free end. The hypotheses on the beam structure and on the load conditions lead to analytical expressions for the position of the center of rotation of the free-end section. Analytical solutions are compared to the results obtained by finite element analysis. A final example shows how the obtained results can be useful in the determination of the pseudo-rigid body model.

2. Large deflection analysis of constant-curvature cantilever beams

Considering a constant-curvature beam, the Euler–Bernoulli beam equation can be written as

$$\frac{d\theta}{ds} = \frac{1}{r} + \frac{M}{EI}, \tag{1}$$

where s is the arc-length coordinate system, $d\theta/ds$ is the rate of change of angular deflection along the beam, r is the radius of curvature of the beam in the non-deflected configuration, M is the internal bending moment, and EI is the bending stiffness (E is the Young's modulus and I is the moment of inertia of the cross section of the beam with respect to the bending axis).

If l is the length of the beam, the previous equation can be rewritten as

$$\frac{d\theta}{ds} = \frac{\mu}{l}, \tag{2}$$

where

$$\mu = \mu_0 + \tilde{\mu} \tag{3}$$

is a non-dimensional term equal to the sum of

$$\mu_0 = \frac{l}{r}. \tag{4}$$