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Singularity-consistent payload locations for parallel manipulators

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ABSTRACT

Parallel manipulators are practically confined to a dramatically small subregion of their workspace due to type II singularities. Around such configurations, the actuator forces tend to infinity and, consequently, the controllability is lost. Therefore, in recent years, growing attention is devoted to develop methods for parallel manipulators to pass through these singular positions while the required actuator efforts remain bounded and continuous. With this aim, in this paper the singularity-consistent payload locations are analytically studied. First, it is shown for three-degree-of-freedom planar parallel manipulators that, in general, the corresponding locus describes a circle in the end-effector plane, whereas it will be a straight line if the angular acceleration of the end-effector platform is prescribed to be zero at the singular configuration. Then, it is proved for spatial six-degree-of-freedom parallel manipulators that the locus of interest represents a quadric surface, whereas it degenerates to a plane if the end-effector platform is prescribed to be in pure translation. The developed payload placement method is outlined as four theorems and one corollary, and its effectiveness is demonstrated through numerical simulations.

1. Introduction

When compared to serial robotic arms, parallel manipulators offer promising advantages like improved acceleration capabilities (due to lower moving inertias as a result of the fact that the actuators can be located at or near the base) and higher payload-to-weight ratio [1,2]. However, the existence of type II singularities, generally within the workspace [3], is seen as one of their biggest disadvantages. In the neighborhood of such singular positions, the required actuator efforts diverge to infinity, and consequently, the actuators saturate. Hence, parallel manipulators are uncontrollable around these singular configurations [4,5], and a desired path passing through such a configuration cannot be realized. Driving the robot in an attempt to cross these singularities may result in its permanent damage as well [6]. For all these reasons, a parallel robot is practically confined to only a dramatically small fraction of its workspace that is determined by the corresponding singularity loci [7–9]. On the other hand, type I singularities mostly occur at the workspace boundaries [3], and for this reason, they are not considered as a serious limitation for parallel manipulators [10]. Therefore, in the current study, only type II singularities are focused on. In the rest of the paper, by *singularity* is meant *type II singularity*.

One straightforward solution to the problem is the use of redundant actuators, which will eliminate singularities [11,12]. However, these additional actuators will indeed bring an increase in cost and control complexity [7].

Another solution is the planning of the trajectory so that the dynamic equations of motion of the manipulator are consistent at the singular position [7,13–15]. In the framework of this method, a given path passing through a singularity is realized by

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imposing singularity-related constraints on the velocities and accelerations of the end-effector. However, as should be apparent, an arbitrarily prescribed trajectory is unrealizable with this strategy [7,13–15].

In this context, to enhance robustness to model uncertainties while crossing the singularities, Pagis et al. [9] suggested a multi-model computed-torque control scheme, provided that some conditions related to the nullification of the degenerating part of the inverse dynamic model are met through proper planning of the trajectory. Parsa et al. [16] applied the concept of using active masses, but this will clearly require a redundant number of actuators, the disadvantages of which are previously given above.

In the current paper, as a solution to the stated problem, a singularity-consistent payload placement technique is proposed. For this purpose, the payload dynamics are studied by the use of the Lagrangian method and the corresponding locations are analytically derived such that the dynamic equations of the loaded manipulator are consistent at the singular position. The developed method is formalized as four theorems and one corollary. Numerical simulation results are presented to demonstrate the implementation and effectiveness of the introduced technique.

2. Method of singularity-consistent payload placement

Consider a non-redundant parallel manipulator with n degrees of freedom (DOFs). The closed-loop mechanism can be converted to an m-DOF open-chain system by disconnecting an enough number of passive joints for removing the loop-closure constraints. Let $\mathbf{q}$ denote the vector of the joint variables of the open-chain system. The dynamic equations of motion of the manipulator can be written as [13,14]:

$$\mathbf{HF} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} \tag{1}$$

where $\mathbf{M} = \mathbf{M}(\mathbf{q})$ is the $m \times m$ generalized inertia matrix and $\mathbf{N} = \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$ is the m-dimensional vector of the generalized Coriolis, centrifugal and gravity forces of the open-chain system. On the left-hand side of Eq. (1), all the unknown actuator and constraint forces are combined into an m-dimensional vector $\mathbf{F}$, and $\mathbf{H} = \mathbf{H}(\mathbf{q})$ denotes the $m \times m$ coefficient matrix of this vector.

At a type II singularity, $\mathbf{H}$ is rank deficient, usually by one [7,14], and the inverse dynamics solution grows unboundedly in its neighborhood. Let it be assumed in the present paper that $\mathbf{H}$ is rank deficient by one at the singular position. Let further kth row of $\mathbf{H}$ be linearly expressed in terms of its other rows at this position, and $c_j = c_j(\mathbf{q})$ denote the coefficient of its jth row ($j = 1, \cdots, m, j \neq k$) in this expression. The consistency condition to be satisfied at the singularity can then be written as [13,14]:

$$R_k = \sum_{j=1\ (j \neq k)}^{m} c_j R_j \tag{2}$$

where $\mathbf{R} = \mathbf{R}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ represents the m-dimensional right-hand side vector of Eq. (1) given by

$$\mathbf{R} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} \tag{3}$$

2.1. The underlying principle of the method

The main motivation of the current study can be summarized as to place the payload such that the dynamic equations of motion of the parallel manipulator are consistent at the singular position. Let the dynamic model of the unloaded manipulator be given by Eq. (1). The dynamic equations of motion of the loaded robot can then be expressed as

$$\mathbf{HF} = \mathbf{R}' \tag{4}$$

where the m-dimensional vector $\mathbf{R}' = \mathbf{R}'(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is

$$\mathbf{R}' = \mathbf{R} + \mathbf{M}^p\ddot{\mathbf{q}} + \mathbf{N}^p \tag{5}$$

Here, $\mathbf{M}^p = \mathbf{M}^p(\mathbf{q})$ is the $m \times m$ inertia matrix of the payload, and $\mathbf{N}^p = \mathbf{N}^p(\mathbf{q}, \dot{\mathbf{q}})$ is the m-dimensional vector that collects all the Coriolis, centrifugal and gravity terms arising in the dynamic equations due to the payload.

Hence, in accordance with Eq. (2), the new consistency condition to be satisfied for the loaded manipulator at the singularity can be written as

$$R'_k = \sum_{j=1\ (j \neq k)}^{m} c_j R'_j \tag{6}$$

The basic principle of the method can equivalently be stated as to ensure at the singular configuration that $\mathrm{rank}([\mathbf{H} \,\vdots\, \mathbf{R}']) = \mathrm{rank}(\mathbf{H}) = m - 1$ through a proper placement of the payload. Note that $[\mathbf{H} \,\vdots\, \mathbf{R}']$ is the $m \times (m + 1)$ augmented matrix for Eq. (4).

Having established the preliminaries, the singularity-consistent payload locations are first derived for 3-DOF planar parallel manipulators in Section 2.2 and then generalized for spatial 6-DOF parallel manipulators in Section 2.3.