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Contact stress calculation of undercut spur and helical gear teeth

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ABSTRACT

The presence of undercut at the pinion root may affect the load sharing among couples of spur gear teeth in simultaneous contact, as well as the load distribution along the line of contact of helical gears. This occurs if the outside points of the wheel profile do not find, due to undercut, mating profile to mesh with, which is called vacuum gearing. Under these conditions, the effective contact ratio is reduced, and the critical contact points may be shifted from their locations at non-undercut profiles. In this paper, a non-uniform model of load distribution along the line of contact, obtained from the minimum elastic potential criterion, has been used combined with the Hertz equation for the calculation of the contact stress. A complete study on the critical load conditions and the value of the critical contact stress has been carried out, considering a wide range of the values of the geometrical parameters. As a result, a recommendation for pitting load capacity calculations of vacuum-gearing spur and helical gears is proposed.

1. Introduction

Calculation methods of spur and helical gear drives for preliminary designs or standardization purposes available in technical literature [1–3] use the Hertz equation to evaluate the contact stress. The load is assumed to be uniformly distributed along the line of contact, and therefore several influence factors for load distribution are introduced to correct the calculated values of the bending and contact stresses [1,4]. In fact, it is known that the load distribution depends on the meshing stiffness of the pair of teeth, which is different at any contact point, which means the load per unit of length is also different at any point of the line of contact.

A model of load distribution for spur and helical involute gears based on the hypothesis of minimum elastic potential energy has been developed by the authors [5,6]. This model has been used to develop more accurate calculation methods of load capacity [7,8] and efficiency [9–11]. However, the numerical method used for the integration of the elastic potential energy provides numerical values of the load per unit of length at discrete contact points and meshing positions. This allows to obtain some conclusions regarding the considered gear pairs, but makes very difficult to extract general conclusions, valid for any gear pair.

Some other studies on the load distribution along the line of contact can be found in technical literature [12–20], but all of them provide results obtained by numerical techniques or the Finite Element Method presenting the same problem of no generality of the obtained results.

Recently, the authors [21] presented an approximate analytic equation for the inverse unitary potential (the inverse of the tooth-pair potential for unit load and face width), very simple and accurate, which depends exclusively on the transverse contact ratio. The load per unit of length can be computed from the inverse unitary potential and its integral along the complete line of contact, but this integral can be easily computed, as the inverse unitary potential has now an analytic expression. This made

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possible general studies on the pitting load capacity [22] and efficiency [23,24], allowing to make proposals for calculation methods for preliminary designs or standardization purposes.

However, undercut teeth were not considered in the mentioned works. Neither researchers nor designers have ever paid much attention to undercut, as its presence weakens the teeth and it is relatively easy to avoid at the design stage. Surely for this reason, technical literature rarely presents results or calculations for undercut teeth [1–3]. Nevertheless, undercut should be taken into account at least for standardization purposes. In some cases undercut arises combined with other desirable effects which improve the tooth behavior, resulting in gear teeth with higher properties. For example, for a given number of teeth, a smaller pressure angle may produce undercut – or increase the amount of undercut – however, the contact ratio increases, so that the load capacity may improve as the load is distributed along a longer line of contact [25].

Reference [21] also presents a corrected equation of the inverse unitary potential to take into account the presence of undercut at the tooth root, which has influence on the load distribution if the outside points of the wheel profile do not find mating profile to mesh with. This phenomenon is called vacuum gearing. From this corrected equation, the authors presented the load sharing model [26] and the critical contact stress calculation method [27] for undercut spur gears.

In this paper, the study is extended to undercut helical gears. The load distribution along the line of contact is determined, and a complete study of the location and the value of the critical contact stress on involute spur and helical gear teeth with transverse contact ratio between 1 and 2, has been carried out. According to this, a proposal for pitting calculations is established. As the effect of vacuum gearing on the contact is very similar to that of a shorter addendum on the wheel teeth, the same approach has been used to study the critical contact stress of spur and helical gears with different addendum on pinion and wheel.

2. Load distribution model

Reference [21] presents in detail the model of load distribution of minimum elastic potential energy. The elastic potential energy U is computed from the equations of the theory of elasticity and the teeth geometry. For calculations, a spur gear with unit load and face width is considered. Its elastic potential u – named unitary potential – and its inverse unitary potential $v = u^{-1}$ are both dependent on the contact point, which is described by the ξ parameter of the contact point at the pinion profile as:

$$\xi = \frac{z_1}{2\pi}\sqrt{\frac{r_{C1}^2}{r_{b1}^2} - 1} \tag{1}$$

where z is the number of teeth, r_C the radius of the contact point, r_b the base radius and subscript 1 denotes the pinion (subscript 2 will denote the wheel). Note that the difference of ξ parameters corresponding to contact at the outer point of contact and at the inner point of contact is equal to the transverse contact ratio ε_α. Similarly, the difference of ξ parameters corresponding to two contiguous teeth in simultaneous contact is equal to 1.

For spur gears, if the elastic potential energy is computed considering all the pairs of teeth in simultaneous contact, with an unknown fraction of the load acting on each one, and minimizing its value by means of variational techniques (Lagrange's method), the load at each pair results in [21]:

$$F_i(\xi_i) = \frac{v_i(\xi_i)}{\sum_{j=0}^{z_1-1} v_j\left(\xi_j\right)} F \tag{2}$$

where F_i (ξ_i) and v_i (ξ_i) are the load and the inverse unitary potential of tooth i when contact occurs at the point corresponding to ξ_i, F is the total transmitted load, and it is assumed $v_i(\xi_i) = 0$ outside the interval of contact. According to this, the load sharing ratio $R(\xi)$ (or the fraction of the load supported by the considered pair of teeth) is given by:

$$R_i(\xi_i) = \frac{F_i(\xi_i)}{F} = \frac{v_i(\xi_i)}{\sum_{j=0}^{z_1-1} v_j\left(\xi_j\right)} = \frac{v(\xi_i)}{\sum_{j=0}^{z_1-1} v(\xi_i + (j-i))} \tag{3}$$

while the load per unit of length $f(\xi)$, for spur gears, can be expressed as:

$$f_i(\xi_i) = \frac{F}{b} R_i(\xi_i) \tag{4}$$

being b the effective face width.

The same approach may be used for helical gears by dividing the helical tooth in infinite slices, perpendicular to the gear axis. Each slice is equivalent to a spur gear with differential face width. In this case, the difference between the ξ parameters of two slices separated a distance dδ along the gear axis (or dl along the line of contact) is:

$$d\xi = \frac{\varepsilon_\beta}{b} d\delta = \frac{\varepsilon_\beta \cos\beta_b}{b} dl \tag{5}$$