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Determination of joint reaction forces in a symbolic form in rigid multibody systems

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ABSTRACT

This paper presents an algorithm for determination of joint reaction forces in a symbolic form in planar and spatial tree structure rigid multibody systems. The frictionless revolute and prismatic joints are taken into consideration. The algorithm is based on the use of Kane's equations with undetermined multipliers of constraints. The expressions for the reaction forces and the torques of reaction couples in the joints are obtained in a form that does not require matrix inversion and allows an easy and straightforward implementation in programming environments for symbolic computations (like Mathematica, Maple etc.). The application of the proposed algorithm to the problem of determination of static friction forces in locked Coulomb friction joints is indicated. The algorithm has been illustrated by using both a gymnast on a trampoline and a Puma manipulator.

1. Introduction

In dynamics of machines and mechanisms, determination of joint reactions represents an important task. The joint reactions are required for the analysis of a stress state in the joints on the basis of which dimensioning of the joint constructive elements is done. Also, to get a more realistic picture of the dynamic behaviour of multibody systems the Coulomb friction in joints needs to be considered. For the determination of the Coulomb friction forces it is also needed to determine the joint reactions (see e.g. [1–12]).

One of the most often applied algorithms for the determination of joint reactions in open kinematic chains was described in [13]. It is based on the use of the Newton–Euler formalism regarding the description of the system dynamics and the formation of the corresponding kinematic and dynamic recursive relations. In [2,3], the improvement of computational efficiency of the method given in [13] has been done by simplifying the expressions for certain quantities by means of symbolic calculations. Such a derived modified Newton–Euler algorithm was used in [2,3] for the determination of friction forces in joints. Screw theory and dual-number matrices are used in [14] for the derivation of the so-called dual-Euler equations. Based on these equations, the differential equations of the motion of open kinematic chains as well as the symbolic expressions for joint reactions have been derived in a symbolic form. The references [1,15] constitute the second group of papers in which the determination of joint reactions are based on the non-recursive Newton–Euler method. Applying this method, the reaction forces in joints in open and closed kinematic chains [15] and in rotative guides with Coulomb friction [1] were determined. In regard to the Coulomb friction in joints it should be emphasised that the reaction forces in frictionless joints and the ideal constraint reactions in Coulomb friction joints are equal under the same kinematic and dynamic conditions as shown in [4]. Furthermore, a method was developed [16–18] for the determination of joint reactions based on the introduction of coordinates corresponding to the prohibited motions in joints and the derivation of pseudo-inverse matrices, where in [16] the closed-loop multibody systems are considered also. In [17,18], the method was applied to the problems of human body

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dynamics. In [19], applying the technique similar to the one of replacing joints with more than one degree of freedom by equivalent combinations of revolute and prismatic joints (see e.g. [20,21]), both the real prismatic and revolute joints were replaced by a system of six joints (prismatic and revolute) among which five are the so-called fictitious joints. In this system the relative motions in the fictitious joints correspond to the motions which are not allowed by real joints. After the derivation of the corresponding equations of motions and their numerical solving the joint constraint loads during optimal dynamic motion of redundant manipulators were determined in [19]. In [9], a problem of the determination of joint reactions in flexible multibody systems was considered.

In this paper, an algorithm for the determination of joint reactions in tree-like multibody structures is proposed by the modelling technique based on fictitious bodies [22] and the general method from [23,24] for determination of constraint reactions. The method from [23] and ([24], pp. 349–353), which is based on Lagrange's equations, is adapted for the use of Kane's equations. Spatial and planar multibody systems are particularly considered. Applications of the algorithm are illustrated through examples.

2. Kinematic description of a system

Consider a system of n rigid bodies interconnected by revolute or prismatic frictionless joints. The multibody system has the form of a tree-like multibody structure which moves in a uniform gravitational field (see Fig. 1(a)). In Fig. 1(a), labelling of the bodies is done as in [25] so the body is connected to the fixed reference base body (V_0) by either a revolute joint or a prismatic joint which has the notation (V_1) while any other body in the system has the subscript greater than the subscript of its preceding adjacent body on a direct path from body (V_1). In this case, the direct path between bodies (V_1) and (V_i) means that such a path passes through the bodies only once. Note that a joint has the same index as the subscript of the following adjacent body in the pair of bodies connected by this joint. Further, an inertial reference frame $Oxyz$ fixed to body (V_0) has the vertical z axis directed upwards. For the numbering procedure the subscripts of bodies satisfy the following relation [25]:

$$1 < 2 < \ldots < j < j+1 < \ldots < p1 < j1 < \ldots < p2 < j2 < \ldots < p3 = n\ . \tag{1}$$

According to the above mentioned description of the system, the following matrices can be introduced [25]:

$$\Theta_{(i)} \in R^{i\times 1},\ \ i = 1,\ldots,n \tag{2}$$

whose components are

$$\Theta_{k,(i)} = \begin{cases} 1, & \text{if body } (V_k) \text{ is in the direct path from } (V_1) \text{ to } (V_i) \\ 0, & \text{otherwise} \end{cases}, \tag{3}$$

where $1 \leq k \leq i$. The other descriptions of tree-like multibody systems are given in [20,26–28]. The considered multibody system has n degrees of freedom and its motion is described by generalised coordinates $q_i (i = 1, \ldots, n)$. In the case of a prismatic joint, the coordinate q_i represents the relative linear displacement of body (V_i) with respect to its preceding adjacent body (V_h) (see Fig. 1(b)) measured along the joint axis determined by the unit vector $\vec{e}_i$, while in the case of a revolute joint this coordinate describes the relative rotation of body (V_i) with respect to (V_h) carried out about the axis $\vec{e}_i$. The vector $\vec{e}_i$ is fixed to

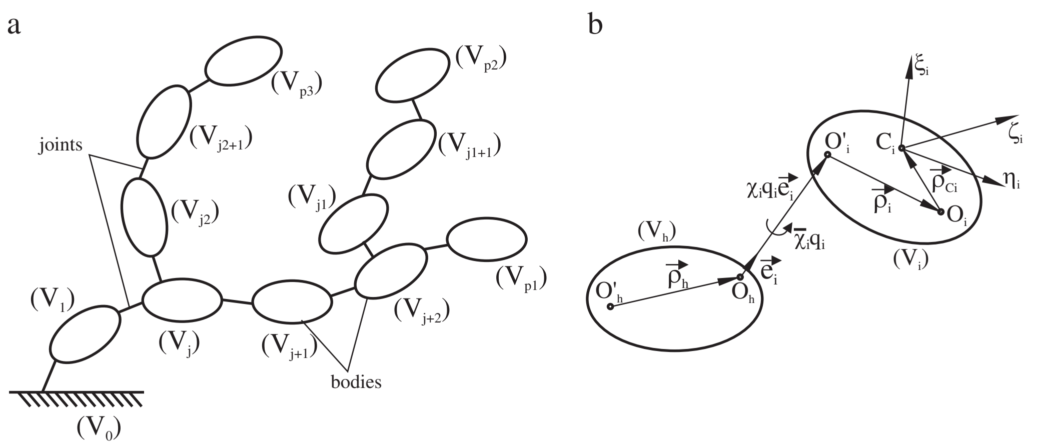

Fig. 1. (a) A tree structure multibody system; (b) kinematics of a joint.