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General network reliability problem and its efficient solution by Subset Simulation

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ABSTRACT

Complex technological networks designed for distribution of some resource or commodity are a pervasive feature of modern society. Moreover, the dependence of our society on modern technological networks constantly grows. As a result, there is an increasing demand for these networks to be highly reliable in delivering their service. As a consequence, there is a pressing need for efficient computational methods that can quantitatively assess the reliability of technological networks to enhance their design and operation in the presence of uncertainty in their future demand, supply and capacity. In this paper, we propose a stochastic framework for quantitative assessment of the reliability of network service, formulate a general network reliability problem within this framework, and then show how to calculate the service reliability using Subset Simulation, an efficient Markov chain Monte Carlo method that was originally developed for estimating small failure probabilities of complex dynamic systems. The efficiency of the method is demonstrated with an illustrative example where two small-world network generation models are compared in terms of the maximum-flow reliability of the networks that they produce.

1. Introduction

Complex technological networks are a pervasive feature of modern society. The worldwide increase in urbanization and globalization, accompanied by rapid growth of infrastructure and technology, has produced complex networks with ever more interdependent components. These networks are designed for distribution of some resource or commodity. Examples include transportation networks (e.g. networks of roads or rail lines, or networks of airline routes), communication networks (e.g. telephone networks or the Internet), and utility networks (e.g. networks for delivery of electricity, gas or water).

Technological networks are so deeply integrated into the infrastructure of megacities that their failures, although rare, often have serious consequences on the wellbeing of society. Societal dependence on technological systems and networks is constantly growing, giving an ever increasing vulnerability to their failure. As a result, there is an increasing demand for modern technological networks to be highly reliable in their operations. The degree to which a network is able to provide the required service needs to be quantitatively assessed during its design and operation, taking into account uncertainty in the future demand, supply and network operational capacity.

Traditional methods for network reliability analyses are based on graph theory and mostly look at small scale networks. These methods aim to exactly compute the network reliability and can be roughly classified by the following (not mutually exclusive) three categories: enumeration methods, direct methods, and decomposition methods. *Enumeration methods* are typically based on either complete state enumeration or more sophisticated methods such as minpath or mincut enumeration, e.g. [1]. *Direct methods* are intended to compute the reliability of a network from the structure of the underlying graph, without a preliminary search for the minpaths and mincuts, e.g. [13]. In *decomposition methods*, the main idea is to divide the network into several subnetworks, and the overall reliability is then calculated based on the reliabilities of the corresponding subnetworks, e.g. [21]. A detailed review of traditional methods for reliability analysis of small scale networks is provided in [15]. All these methods in one way or another are based on combinatorial exhaustive search through the network.

On the other hand, one of the inherent characteristic features of modern technological networks is their very large size. Today the complexity of real-world networks can reach millions or even billions of vertices and edges with incomprehensible topology. Fig. 1 shows a visual representation of a small portion

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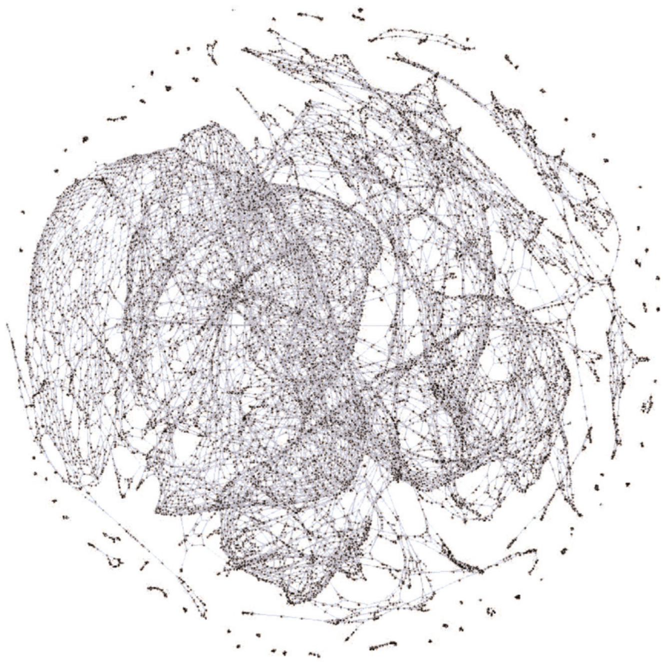

Fig. 1. Man-made "galaxy": a visual representation of a small portion (~1%) of a California road network. Intersections and road endpoints are represented by vertices and the roads connecting these intersections or endpoints are represented by undirected edges. The network data is available for free at http://snap.stanford.edu/data/roadNet-CA.html. Visualization was done using the Network Workbench Tool (available for free at http://nwb.slis.indiana.edu).

(approximately 1%) of a California road network. In this network, intersections and road endpoints are represented by vertices and the roads connecting these intersections or endpoints are represented by undirected edges.

This dramatic change of scale induces a corresponding change in the philosophy of reliability analyses. Many of the exhaustive search algorithms that have been applied to small networks are simply not feasible for large networks, since essentially all reliability problems of interest are NP-hard [27] and the exhaustive algorithms grow in complexity very rapidly as a function of the network size. It has been thus recognized that the classical methods of reliability and risk analysis fail to provide the proper instruments for analysis of actual modern networks [32]. As a result, a new field of research has recently emerged with the focus shifting away from the combinatorial exhaustive search methodology to the study of statistical properties of large networks, together with the study of their robustness to random failures, errors, and intentional attacks.

In this paper, we propose a stochastic framework for quantitative assessment of network reliability in the presence of uncertainty, formulate a general network reliability problem within this framework, and show how to solve this problem using Subset Simulation [5], an efficient Markov chain Monte Carlo method that was originally developed for estimating small failure probabilities of complex dynamic systems, such as civil engineering structures at risk from earthquakes. The new theory was first presented in the conference paper [36] but here we give a fuller explanation and extended results. We remark that Subset Simulation has also been used previously for evaluating origin–destination connectivity reliability of lifeline networks [9].

We proceed as follows. In the next section, we highlight the similarity between reliability problems for complex systems and complex networks, and formulate a general network reliability problem subjected to several realistic conditions that make this problem computationally difficult. In Section 3, we describe the Subset Simulation algorithm for solving the network reliability problem. An illustrative example that demonstrates how Subset Simulation can be effectively used for solving the maximum-flow reliability problem and for finding reliable network topologies is provided in Section 4. Concluding remarks are made in Section 5.

2. From complex systems to complex networks

Complex networks are often viewed as the structural skeletons of complex dynamic systems. While networks are a relatively new object of study in reliability engineering, the reliability of dynamic systems is a well-established and deeply researched problem. The engineering research community has developed several very efficient methods for estimation of reliability of complex dynamic systems such as tall buildings, bridges, and aircraft [4,5,18,7,8,16,34,35]. Moreover, it can be shown (see, e.g. [33]) that the system reliability problem is mathematically equivalent to two other extensively researched problems: finding the free energy of a physical system (statistical mechanics), and finding the marginal likelihood of a Bayesian statistical model (Bayesian statistics). All three problems can be considered as the problem of estimating the ratio of normalizing constants for a pair of probability distributions.

As a first step towards efficient network reliability methods, this paper focuses on the development of a network analog of the system reliability method, Subset Simulation [5]. In Section 2.1, we briefly review the system reliability problem to demonstrate its similarity with the network reliability problem which is discussed in Section 2.2.

2.1. System reliability problem

Calculation of the reliability, or equivalently the probability of failure p_F, of a dynamic system under given excitation conditions is one of the most important and challenging problems in reliability engineering. The uncertainty in the input excitation $x \in \mathbb{R}^m$ is quantified by a joint probability density function (PDF) $\pi(x)$. The performance of the dynamic system under this input is quantified by a *performance function* $\mu: \mathbb{R}^m \to \mathbb{R}$ through a dynamic input–output model of the system. For example, if our system corresponds to a tall building, the input x may represent an uncertain earthquake excitation sampled at discrete times over some interval and the performance $\mu(x)$ may represent the corresponding maximum roof displacement over this duration, or the maximum interstory drift over all stories for the duration, calculated from the dynamic model.

Define the failure domain $F \subset \mathbb{R}^m$ as the set of inputs ("failure points") that lead to the exceedance of some prescribed critical threshold $\mu^* \in \mathbb{R}$:

$$F = \{x \in \mathbb{R}^m | \mu(x) > \mu^*\} \quad (1)$$

In the above example, the critical threshold μ^* represents the maximum permissible roof displacement or maximum permissible interstory drift and so the failure domain F represents the set of all earthquake excitations that lead to unacceptable deformation of the tall building.

The *system reliability problem* is then to compute the probability of failure that is given by the following integral:

$$p_F = \mathbb{P}(x \in F) = \int_F \pi(x)\,dx = \int_{\mathbb{R}^m} \pi(x) I_F(x)\,dx = \mathbb{E}_\pi[I_F], \quad (2)$$

where $\mathbb{E}_\pi$ denotes expectation with respect to the distribution $\pi(x)$ and I_F is the indicator function of the failure domain F: $I_F(x) = 1$ if the system subject to excitation x fails (i.e. the output $\mu(x)$ is not acceptable according to the performance criterion, $\mu(x) > \mu^*$) and