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A new system formulation for the tolerance analysis of overconstrained mechanisms

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ABSTRACT

The goal of tolerance analysis is to verify whether design tolerances enable a mechanism to be functional. The current method consists in computing a probability of failure using Monte Carlo simulation combined with an optimization scheme called at each iteration. This time consuming technique is not appropriate for complex overconstrained systems. This paper proposes a transformation of the current tolerance analysis problem formulation into a parallel system probability assessment problem using the Lagrange dual form of the optimization problem. The number of events being very large, a preliminary selective search algorithm is used to identify the most contributing events to the probability of failure value. The First Order Reliability Method (FORM) for systems is eventually applied to compute the probability of failure at low cost. The proposed method is tested on an overconstrained mechanism modeled in three dimensions. Results are consistent with those obtained with the Monte Carlo simulation and the computing time is significantly reduced.

1. Introduction

A manufacturing process is not able to provide exactly the same workpieces; indeed, theoretical dimensions of a design product cannot be reached in a repetitive manner (tool wear, operator variability, etc.). The mechanism behavior is disturbed by geometrical deviations as well as gaps between different parts of the mechanism. Design tolerances are therefore specified on different features of the mechanism to limit the deviations. Tolerance analysis aims at analyzing the impact of these admissible variations on the mechanism behavior. The main stake is to evaluate a quality level of the product during its design stage. The technique used consists of assessing a probability of failure P_f of the mechanism of magnitude around 10^{-6} for large series production. This value represents the probability that a functional condition, $C_f = y_{th} - Y \geq 0$, is not satisfied, where Y is a functional characteristic of the mechanism and y_{th} is a threshold value to not be exceeded.

Tolerance analysis methods must consider the geometrical deviations as random variables whose probabilistic distributions are chosen regarding the manufacturing process [1,2]. However, gaps between parts or contact points cannot be modeled by aleatory uncertainty. Gaps belong to the parameter uncertainty category [3] of the epistemic uncertainty which makes difficult the mechanical behavior of this kind of mechanisms to be modeled. Indeed, analyzing isoconstrained or overconstrained mechanisms is different. An assembly which have only its six degrees of freedom fixed in three dimensions (three degrees of freedom in two dimensions) is considered to be an isoconstrained mechanism, usually without gaps. On the contrary, an assembly which have more than six degrees of freedom fixed is considered as an overconstrained mechanism. Gaps allow this kind of mechanism to be assembled although more than six degrees of freedom are fixed. Fig. 1 shows a simple isoconstrained mechanism in one dimension where the functional characteristic Y must not exceed a specified threshold. On such a mechanism, the expression of this characteristic Y is a function only of the dimensions x_1 and x_2. This kind of problem is well-defined.

In contrast, Fig. 2 shows two configurations of an overconstrained mechanism. Now the functional characteristic Y is a function of the random variables and of the gap values. However, following the realization of the random variables, gap values are depending on the location of the contact point between part 1 and part 2. In this case the tolerance analysis problem is

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Nomenclature

P_f	functional failure probability
y_{th}	threshold value
X	geometrical deviation = random variable
g	gap variable = optimization variable
$C_f \geq 0$	functional condition
$C_{f,dual} \geq 0$	functional condition in the dual form
$C \leq 0$	interface constraints
N_C	number of interface constraints
N_s	number of possible situations
$N_{as} \leq N_s$	number of admissible situations
$N_{ds} \leq N_{as}$	number of dominant admissible situations

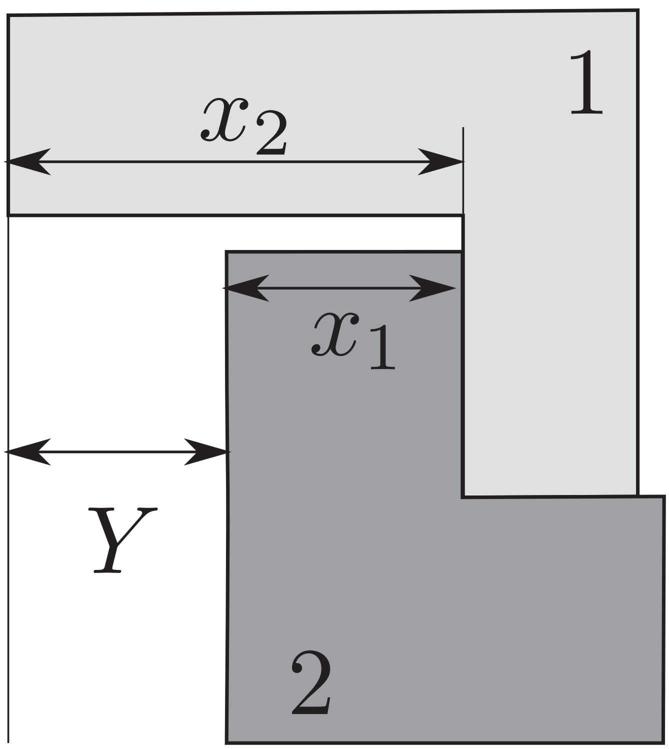

Fig. 1. Isostatic mechanism. The functional characteristic Y can be expressed as a function of variables x_1 and x_2.

overconstrained due to the multiple possible configurations of gaps. Tolerance analysis methods must therefore take into account the worst configurations of gaps to compute the probability of failure. This operation is usually performed using an optimization scheme [1]. This particularity turns out to deal with a system probability assessment, the transition from one worst configuration to another leading to an abrupt change of the limit-state surface.

For an isoconstrained mechanism, computing the probability of failure is simple because the functional condition is only expressed as a function of the random variables which describes the geometrical parameters. Classical reliability methods such as Monte Carlo simulation or variance reduction techniques like importance sampling [4] can be used to quickly compute the probability of functional failure. For complex non-explicit applications, surrogate models replacing the true functional condition may also be used in simulations in order to save time. Numerous techniques used in computer experiments exist in the literature such as quadratic response surfaces [5,6], Kriging [7,8], support vector machines [9–11] and polynomial chaos [12–14]. Approximation methods like FORM [15,16] can also be performed. All methods are efficient provided that the problem has a smooth limit-state surface which is the case for an isoconstrained mechanism tolerance analysis problem. For an overconstrained mechanism, these techniques, except the Monte Carlo simulation, can no longer be used because of a piecewise limit-state function coming from the different configurations of gaps.

This paper intends to propose an efficient method to compute the probability of failure of a tolerance analysis problem in the case of overconstrained mechanisms. The technique is based on a transformation of the tolerance analysis problem formulation using the Lagrange duality property. This operation leads to an auxiliary problem which is free from the optimization step. The solution method includes a selective search algorithm so as to determine the dominant failure situations among the numerous possible ones. The probability of failure is eventually computed using the First Order Reliability Method (FORM) for systems [15,16].

The paper is organized as follows: Section 2 shows the current tolerance analysis problem formulation whose probability is estimated thanks to the Monte Carlo simulation. Section 3 describes the mathematical transformation of the optimization problem into its Lagrange dual. Section 4 is devoted to the comparison of the proposed method with the Monte Carlo simulation on different applications: first, the different transformation steps of the proposed formulation are detailed on a simple academic example. Then the method is applied to an overconstrained industrial application modeled in three dimensions.

2. Tolerance analysis of overconstrained mechanisms

2.1. Problem formulation based on quantifiers

The presence of gaps in overconstrained mechanisms makes the mechanical behavior difficult to model. Gaps are considered as free variables, but they are not free of constraints because interpenetration between two surfaces of two parts of the mechanism cannot be allowed. A set of N_C interface constraints are therefore defined to prevent surfaces from penetrating into each other. Let $\mathbf{X} = \{X_1, \ldots, X_n\}$ be the vector of random variables and $\mathbf{g} = \{g_1, \ldots, g_m\}$ the vector of gaps. Given a realization $\mathbf{x}$ of the random vector $\mathbf{X}$, these constraints are inequations written as follows:

$$\{C_k(\mathbf{x}, \mathbf{g}) \leq 0\}_{k=1,\ldots,N_C} \quad (1)$$

The functional condition equation of the mechanism is expressed as follows:

$$C_f(\mathbf{x}, \mathbf{g}) = y_{th} - Y \geq 0 \quad (2)$$

where $Y = f(\mathbf{x}, \mathbf{g})$ is the response of the system (a parameter such as a gap or a functional characteristic) modeled by a function f characterizing the influences of the deviations and gaps on the mechanism behavior [17].

The universal quantifier "∀" (all) is used to translate the concept that the functional condition must be respected in all configuration of the mechanism. The definition of the functionality of the mechanism is given by Qureshi et al. [1]: "for all admissible gap configurations of the mechanism, the geometrical behavior and the functional requirement are respected". For any realization