Probabilistic Engineering Mechanics 37 (2014) 16–23

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Stochastic analysis of the critical velocity of an axially moving cracked elastic plate

M. Tirronen [a,*], N. Banichuk [a,b], J. Jeronen [a], T. Saksa [a], T. Tuovinen [a]

[a] Department of Mathematical Information Technology, University of Jyväskylä, P.O. Box 35 (Agora), FI-40014, Finland
[b] Institute for Problems in Mechanics RAS, Prospect Vernadskogo 101, Bld. 1, 119526 Moscow, Russian Federation

ARTICLE INFO

Article history:
Received 4 November 2013
Accepted 4 April 2014
Available online 13 April 2014

Keywords:
Uncertainty
Fracture
Stability
Moving material
Plate
Paper

ABSTRACT

In this study, a probabilistic analysis of the critical velocity for an axially moving cracked elastic and isotropic plate is presented. Axially moving materials are commonly used in modelling of manufacturing processes, like paper making and plastic forming. In such systems, the most serious threats to runnability are instability and material fracture, and finding the critical value of velocity is essential for efficiency. In this paper, a formula for the critical velocity is derived under constraints for the probabilities of instability and fracture. The significance of randomness in different model parameters is investigated for parameter ranges typical of paper material and paper machines. The results suggest that the most significant factors are variations in the crack length and tension magnitude.

1. Introduction

In industry, there are many systems the behaviour of which can be described by the mathematical model of an axially moving material. Thus, during the last few decades, the mechanics of such materials have aroused much interest among researchers. Traditionally, the studies of axially moving materials are based on a deterministic approach, although in reality, the problem parameters are not known deterministically. In industrial paper manufacturing, which is one of the application areas of axially moving materials, uncertainty factors include, e.g., the strength of the paper web, variation of tension with respect to space and time in the press system, and defects, which vary in their geometry and location in the web. These factors are considerable: according to Uesaka [1], the majority of web breaks in paper production are caused by tension variations, combined with strength variations of the paper web. Wathén [2] discusses the effect of flaws of paper on web breaks and, according to him, even a seemingly flawless paper can fail at very low tensions due to stress concentrations caused by discontinuities, e.g., cuts and shives, in structure.

Finding the optimal value of velocity for an axially moving material is essential, when the efficiency of the corresponding manufacturing process is considered. The most critical threats to good runnability of such a system are instability and material fracture, and on these phenomena a change in the tension magnitude has opposite effects. An increase in tension has a stabilizing effect [3], but high tension may lead to growing or arising of cracks. Web tension too low or too high may cause a web break, which deteriorates production efficiency.

The modelling of vibrations of travelling elastic materials has interested many researchers. The first paper on the subject dates from 1897, when Skutch published a paper [4] concerning the axially moving string. The first papers in English were published in the 1950s, when Sack [5] and Archibald and Emslie [6] studied the axially moving string model. Since then, many researchers have continued the studies of moving elastic material. E.g., Wickert and Mote [7] studied the stability of axially moving strings and beams using modal analysis and Greens function method. The stability of travelling two-dimensional rectangular membranes and plates has been studied, e.g., by Lin [8] and Banichuk et al. [3]. A more extensive literature review of the history of the studies concerning deterministic elastic models can be found in [3], the results of which we also exploit in this study. In the recent studies concerning axially moving plates, material properties such as orthotropicity [9,10] or viscoelasticity [11,12] have been taken into consideration and their effects on the plate behaviour have been investigated.

In addition, there are studies considering stationary plates with random parameters. For example, the free transverse vibrations of elastic rectangular plates with random material properties were considered and statistical characteristics of the random eigenvalues were determined by Sobczyk [13]. Wood and Zaman [14] considered a collection of elastic rectangular plates with random

* Corresponding author.
E-mail addresses: maria.j.e.tirronen@jyu.fi (M. Tirronen), juha.jeronen@jyu.fi (J. Jeronen), tytti.saksa@jyu.fi (T. Saksa), tero.tuovinen@jyu.fi (T. Tuovinen).

http://dx.doi.org/10.1016/j.probengmech.2014.04.001
0266-8920/

inhomogeneities vibrating freely under simply supported boundary conditions. Soares [15] considered uncertainty modelling of plates subjected to compressive loads.

The field of fracture mechanics was developed by Irwin [16], based on the early papers of Inglis [17], Griffith [18] and Westergaard [19]. Various deterministic analyses of vibrations and stability of stationary cracked beams and plates exist in the literature. For a literature review, we refer to [20].

As far as the authors know, axially moving materials have been studied in a stochastic setup only in [21,22]. In these studies, the critical velocity of an axially moving plate was derived in the case in which there is a random length crack on the plate, or the tension, to which the plate is subjected, varies randomly. This research extends these studies by introducing several other parameters as random variables simultaneously in the model. In this paper, we also compare the effect of introducing variation between different problem parameters, in order to decide the randomness of which parameters is the most significant in terms of the critical velocity. For the analysis, we have chosen the setup and parameter ranges to be applicable for paper material and paper making.

The formula for the critical velocity of the plate is derived under constraints for instability and fracture. Depending on the distributions of the problem parameters, numerical methods may be needed in solving the critical velocity. In the paper industrial example, simultaneously introducing several problem parameters as random variables leads to the use of numerical methods. Due to its simplicity and accuracy, we use Monte Carlo simulation to solve the problem with several random variables.

2. Critical velocity of a travelling plate

We consider a rectangular part of an elastic and isotropic band, which is moving at a constant velocity V_0 between supporting rollers. Denoting the part as

$$\mathcal{D} = \{(x,y): \ 0 < x < \ell, \ -b < y < b\}, \tag{1}$$

where ℓ and b are prescribed parameters of length and width, the plate is assumed to travel in the x direction. The supporting rollers are located at $x=0$ and $x=\ell$. (See Fig. 1.)

The considered part $\mathcal{D}$ is represented as a thin elastic plate having constant thickness h, Poisson ratio ν, Young modulus E, and bending rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)}. \tag{2}$$

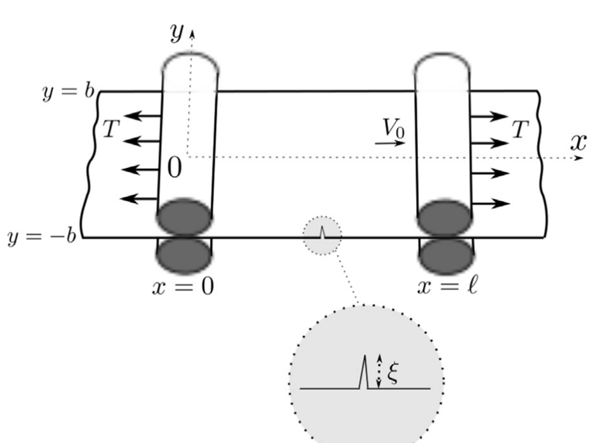

Fig. 1. A travelling elastic plate with a crack.

The mass of the plate per unit area is denoted by m. It is further assumed that the plate is subjected to homogeneous tension T acting in the x direction.

We consider the case in which there is a single crack in the plate. The length of the crack is denoted by ξ. (See also Fig. 1.)

2.1. Characterization of instability of the plate

We first briefly present a deterministic stability analysis for a travelling plate without a crack. Especially, we are interested in critical regimes, where the plate approaches its maximum stable velocity. Details of the analysis can be found in [3].

We perform a standard dynamic analysis (see, e.g., [23]). The transverse displacement of the travelling plate is described by the deflection function w, which depends on the space coordinates x, y and time t. It is assumed that the absolute values of the deflection function w and its derivatives are small. The Kirchhoff plate theory is applied. To study the dynamic behaviour of the plate, the following equation for the travelling plate is used:

$$\frac{\partial^2 w}{\partial t^2} + 2V_0 \frac{\partial^2 w}{\partial x\, \partial t} + (V_0^2 - C^2)\frac{\partial^2 w}{\partial x^2} + \frac{D}{m}\Delta^2 w = 0, \tag{3}$$

where

$$C = \sqrt{\frac{T}{m}} \quad \text{and} \tag{4}$$

$$\Delta^2 w = \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2\, \partial y^2} + \frac{\partial^4 w}{\partial y^4}. \tag{5}$$

As boundary conditions, the classical simply supported and free boundary conditions [24,25] are used. The simply supported boundary conditions read as

$$(w)_{x\,=\,0,\ell} = 0, \quad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x\,=\,0,\ell} = 0, \quad -b \le y \le b, \tag{6}$$

and the equations for the boundaries free of tractions can be presented as follows:

$$\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)_{y\,=\,\pm b} = 0, \quad 0 \le x \le \ell, \tag{7}$$

$$\left(\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2\, \partial y}\right)_{y\,=\,\pm b} = 0, \quad 0 \le x \le \ell. \tag{8}$$

The solution of the dynamic boundary value problem of (3)–(8) can be represented as

$$w(x,y,t) = W(x,y)e^{i\tilde{\omega}t} = W(x,y)e^{\tilde{s}t}, \tag{9}$$

where $\tilde{\omega}$ is the frequency of small transverse vibrations and $\tilde{s} = i\tilde{\omega}$ is the complex characteristic parameter; $\tilde{s} = \operatorname{Re}\tilde{s} + i \operatorname{Im}\tilde{s}$.

If the parameter $\tilde{s}$ is purely imaginary and $\tilde{\omega}$ is real, the plate performs harmonic vibrations of a small amplitude and its motion can be considered stable. If the real part of $\tilde{s}$ becomes positive, the transverse vibrations grow exponentially and, consequently, the behaviour is unstable (see Fig. 2). It can be shown by dynamic analysis that the travelling plate undergoes divergence instability at a sufficiently high speed [8] and thus it is sufficient to perform static analysis, i.e., study the case with $\tilde{s}=0$. (See also Fig. 2.)

The stationary equations for W are, substituting (9) into (3) and setting $\tilde{s}=0$,

$$(mV_0^2 - T)\frac{\partial^2 W}{\partial x^2} + D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2\, \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) = 0 \tag{10}$$

with boundary conditions (6)–(8). We rewrite (10) as

$$-\frac{\ell^2}{\pi^2}\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2\, \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) = \lambda \frac{\partial^2 W}{\partial x^2}, \tag{11}$$