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Modal decomposition using multi-channel response measurements

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ABSTRACT

In this paper a novel algorithm is presented for modal decomposition using multiple channels of measurements of dynamical systems. The algorithm is operated in a two-stage manner. In the first stage, the measurement noise and modeling error are filtered out to obtain the maximum common components, which turn out to be identical to the principal components. However, these maximum common components are not the modal coordinates because it is usually impossible to measure all degrees of freedom. Therefore, the partial mode shape matrix does not possess any orthogonality condition. As a result, the maximum common components will be transformed to the modal coordinates in the second stage using band-pass filter and principal component analysis. The proposed method is computationally very efficient and it does not require a finite element model of the dynamical system. Two simulated examples are presented to demonstrate the efficiency and robustness of the proposed algorithm. Finally, an application using the acceleration field measurements from the Canton tower in Guangzhou is presented.

1. Introduction

Response time series analysis is an important task in various disciplines of science and engineering and it has attracted substantial attention for decades. To resolve the shortcomings of the standard discrete Fourier transform, a number of well-known transformations have been developed. Examples include the short-time Fourier transformation [22] that considers a moving window for the Fourier transform. The wavelet transformation [29,15,17] allows for multi-resolution analysis. Instead of direct transformation techniques, a traditional approach is to analyze the nonstationary response with system identification techniques assuming a prescribed model. Examples include the Kalman filtering [41,18,6,25,40], approaches using extra model constraints [4,37], Bayesian approach [2,36,35,39], particle filter approach [9], and the channel-expansion technique [8].

Blind source separation [3] has received increasing attention in recent years. Since the nature of this problem is underdetermined, different extra conditions need to be imposed, e.g., independence or sparsity. As a result, various types of methods have been developed and it includes the principal component analysis [27,28,5], independent component analysis [13], and non-negative matrix factorization [10], etc. Modal decomposition can be viewed as a branch of blind source separation. Mallat and Zhang [19] proposed the matching pursuits algorithm to decompose signals into linear combination of a dictionary of redundant Gabor functions. The empirical mode decomposition [12] is one of the most popular methods in this area. It provides a direct approach to decompose the measured signal into intrinsic mode functions. Then, the intrinsic mode functions can be analyzed with various tools, e.g., the Hilbert–Huang transform to obtain the instantaneous frequency of the signal [12,34]. This has been used for a number of applications in structural dynamics problems, including structural damage detection [32,33], representation of earthquake ground motion [30], adaptive structural vibration control [21] and signal compression [11], etc. On the other hand, Chen and Kareem [7] proposed a complex modal decomposition approach for coupled buffeting response of bridges. This frequency-domain approach enhances the computational efficiency by avoiding system matrix inversion at each frequency. Wu and Huang [31] proposed the ensemble empirical mode decomposition. It utilizes the artificially added white noise to provide a uniform reference frame in the time–frequency space. By doing so, subjective criterion selection in the intermittence test for the original empirical mode decomposition algorithm can be eliminated.

In this paper, a novel algorithm is proposed for modal decomposition using multiple channels of measurements. The proposed method is a two-stage approach and it does not require a finite element model of the dynamical system. In the first stage, the maximum common components are obtained. They will be shown identical to the principal components so they can be obtained efficiently with simple eigenvalue analysis of small size. Measurement noise and modeling error can be filtered out at this stage. In the second stage, the maximum common components obtained

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in the previous stage are converted to the modal coordinates. It will be shown that this stage is necessary because the maximum common components/principal components are not the modal coordinates. This is due to the limited observed degrees of freedom so the orthogonality does not apply to the partial mode shape matrix.

The structure of this paper is outlined as follows. In Section 2, a simple example is used to introduce the underlying concept of the proposed modal decomposition algorithm. Problems encountered in direct application of this concept for practical circumstances will be presented. In Section 3, the proposed method is presented for modal decomposition using multiple channels of measurements. In the first stage, the maximum common components, which are identical to the principal components, are extracted. In the second stage, the modal coordinates are computed from the maximum common components. Finally, a summary of the proposed algorithm is given. In Section 4, three examples are presented to illustrate the effectiveness and robustness of the proposed method. The first two examples are selected to cover cases with simulated signals so that the true modal coordinates are available for comparison. Therefore, the accuracy and robustness of the proposed algorithm can be assessed. Finally, acceleration field measurements of the Canton tower in Guangzhou are used to illustrate the application in a real world example.

2. Basic concepts and difficulties

In this section, a simple example of two cases is used to illustrate the underlying concept of the proposed method. In Case 1, we introduce the concept of maximum common components, which will be shown identical to principal components in Section 3. It will be demonstrated that the maximum common components can be used for modal decomposition. However, in Case 2, the problem in applying the maximum common components or the principal components for modal decomposition in general situation will be illustrated.

2.1. Case 1

2.1.1. Consider two measured signals with $t \in [0,2\pi]$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathbf{\Phi}\boldsymbol{\xi} = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} \sin t \\ \sin 2t \end{bmatrix} = \begin{bmatrix} 4 \sin t + \sin 2t \\ 2 \sin t - 2 \sin 2t \end{bmatrix} \tag{1}$$

These signals are linear combinations of two sinusoidal functions (regarded as modal coordinates herein) with frequencies 1 and 2 rad/s, respectively. From the measurement of $x_1(t)$ and $x_2(t)$ in the time interval $[0, 2\pi]$, we attempt to reconstruct the two modal coordinates $\boldsymbol{\xi}$, i.e., $\sin t$ and $\sin 2t$. In order to achieve this goal, the maximum common components, denoted by $y(t)$, are extracted from these two signals such that the residuals defined below are minimized:

$$\begin{aligned} \varepsilon_1(t) &= x_1(t) - \phi_1 y(t) \\ \varepsilon_2(t) &= x_2(t) - \phi_2 y(t) \end{aligned} \tag{2}$$

The unknown parameters ϕ_1 and ϕ_2 are introduced to take into account the different contribution of the maximum common component to the two channels of measurements. In order to obtain the optimal solution for ϕ_1, ϕ_2 and $y(t)$, one can minimize the following objective function:

$$J(y,\phi_1,\phi_2) = \int_0^{2\pi} I(y,\phi_1,\phi_2)dt \equiv \int_0^{2\pi} \{[x_1(t)-\phi_1 y(t)]^2 + [x_2(t)-\phi_2 y(t)]^2\}dt \tag{3}$$

It is obvious that there are infinitely many solutions for ϕ_1, ϕ_2 and $y(t)$ because any combination of $\alpha\phi_1$, $\alpha\phi_2$ and $y(t)/\alpha$ gives identical extraction results for nonzero α. Therefore, the solution for $\phi_1 y(t)$and $\phi_2 y(t)$ will be considered instead of individual values for ϕ_1, ϕ_2 and $y(t)$. The underlying concept of the maximum common components is to be extracted such that the residuals are minimized and this is different from the principal component analysis to project the measurements to the principal directions such that the 2-norm of the projection is maximized [26,13]. However, it will be shown in the next section that the maximum common components and the principal components are equivalent.

By using the calculus of variation, the optimal solution can be obtained by taking the variation of the objective function to be zero, i.e., $\delta J = 0$. This is equivalent to solving: $(\partial I(y,\phi_1,\phi_2)/\partial y) = 0$, where the function $I(y,\phi_1,\phi_2)$ is defined in Eq. (3), and the following relationship is readily obtained:

$$y^*(t) = \frac{1}{\phi_1^2+\phi_2^2}[(4\phi_1+2\phi_2)\sin t + (\phi_1-2\phi_2)\sin 2t] \tag{4}$$

Substituting $y^*(t)$ back to Eq. (3), the objective function can be simplified as:

$$J(y^*,\phi_1,\phi_2) = \frac{\pi}{(\phi_1^2+\phi_2^2)}(8\phi_1^2 - 12\phi_1\phi_2 + 17\phi_2^2) \tag{5}$$

Then, the values of ϕ_1 and ϕ_2 can be obtained by solving $(\partial J(y^*,\phi_1,\phi_2)/\partial\phi_j) = 0, j = 1,2$ and it turns out that the two equations are merged to: $(\phi_1-2\phi_2)(2\phi_1+\phi_2) = 0$. Therefore, $\phi_1 = 2\phi_2$ or $\phi_1 = -\phi_2/2$. As a result, there are two sets of solutions:

$$\phi_1^* y^*(t) = 4 \sin t \text{ and } \phi_2^* y^*(t) = 2 \sin t \tag{6}$$

$$\phi_1^* y^*(t) = \sin 2t \text{ and } \phi_2^* y^*(t) = -2 \sin 2t \tag{7}$$

so the modal coordinates in $\boldsymbol{\xi}(t)$ can be recovered from the maximum common components (or equivalently the principal components) in this case. However, the success of this example relies on the orthogonality of the matrix $\mathbf{\Phi}$ in Eq. (1), i.e., $\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I}$, the identity matrix. It will be shown next that this is usually not the case in practice.

2.2. Case 2

In the second case, the same example will be repeated with a different matrix $\Phi = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$. Following the same procedure, the maximum common components are obtained:

$$\begin{aligned} \phi_1^* y^*(t) &= \frac{(6\pm\sqrt{85})}{(6\pm\sqrt{85})^2+7^2}[(38\pm 4\sqrt{85})\sin t + (-1\pm\sqrt{85})\sin 2t] \\ \phi_2^* y^*(t) &= \frac{7}{(6\pm\sqrt{85})^2+7^2}[(38\pm 4\sqrt{85})\sin t + (-1\pm\sqrt{85})\sin 2t] \end{aligned} \tag{8}$$

In this case, the two modal coordinates cannot be decomposed. Unfortunately, this is usually encountered in practice. First, for monitoring projects, it is very rare that the full measurements at all degrees of freedom can be obtained. Therefore, the vectors in the matrix Φ in Eq. (1) contain only some elements of the mode shapes so the well-known orthogonality conditions are not applicable for this partial mode shape matrix. Second, even when full measurements at all degrees of freedom are available, the column vectors $\phi_1, \ldots, \phi_M$ will follow $\phi_m^T \mathbf{M} \phi_{m'} = 0$, if $m \neq m'$ where $\mathbf{M}$ is the mass matrix of the dynamical system. However, $\phi_m^T \phi_{m'} \neq 0$ in general. As a result, if the mass matrix is known, the aforementioned procedure (or the principal component analysis) will have to be modified for a mass matrix weighted inner product in order to perform the modal decomposition. In the next section, a novel decomposition method, which is inspired from the underlying concept of this example, will be presented to overcome these difficulties.