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An efficient simulation approach for multivariate nonstationary process: Hybrid of wavelet and spectral representation method

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ABSTRACT

Currently, the classical spectral representation method (SRM) for nonstationary process simulation is widely used in the engineering community. Although this scheme has the higher accuracy, the time-dependent spectra results in unavailability of fast Fourier transform (FFT) and thus the simulation efficiency is lower. On the other hand, the approach based on stochastic decomposition can apply FFT in the simulation. However, the algorithm including the fitting procedure is relatively complicated and thus limits its use in practice.

In this paper, the hybrid efficient simulation method is proposed for the vector-valued nonstationary process, which contains the spectra decomposition via wavelets and SRM. This method can take advantage of FFT and is also straightforward to engineering application. Numerical examples are employed to evaluate the proposed method. Results show that the method performs fairly well for the scalar process and vector-valued process with real coherence function. In the case of complex coherence function, the majority of the phase in the coherence function cannot be remained in the simulation. In addition, the validity of proper orthogonal decomposition (POD) in nonstationary process simulation via the decomposition of the time-dependent nonstationary spectra is studied. Analysis shows that the direct use of POD in nonstationary spectra decomposition may not be useful in nonstationary process simulations.

1. Introduction

With the increase of computer power, Monte Carlo simulation is widely used in the engineering community to consider the nonlinearity, system stochasticity, stochastic stability and so on. Among the simulation techniques such as time series approach (e.g., [24]) and wavelet approach (e.g., [27]), the spectral representation method (SRM) (e.g., [21,9]), which is based on the spectral representation theorem, may be most popular in engineering practice due to its accuracy and being easy to application. SRM-based simulations of the stationary Gaussian processes for both univariate and multivariate cases were well-developed and summarized in literature (e.g., [21,4]). Thanks to the application of fast Fourier transform (FFT) (e.g., [25]), the simulation is very efficient for the stationary process.

However, the SRM-based stochastic simulation of nonstationary processes requires cumbersome summation of the trigonometric items and thus suffers from the lower efficiency due to unavailability of direct use of FFT (e.g., [5]). The reason is attributed to the fact that the nonstationary power spectra density (PSD), or widely-accepted evolutionary PSD (EPSD), are time-dependent. Also this trait renders EPSD matrix decomposition time-consuming, because the decomposition is required on both time and frequency domains. To improve the simulation efficiency, a few attempts have been conducted. Based on the concept of stochastic decomposition, Li and Kareem [16] suggested a scheme to simulate the multivariate nonstationary process making use of FFT. Li and Kareem [17] also proposed the hybrid discrete Fourier transform and digital filter approach to increase simulation speed. Apart from attempts in use of FFT, the improvement of Cholesky decomposition with PSD/EPSD matrix also could further save the computational resources. Different approaches such as the closed-form formula, interpolation, separation of the phase component and combination of them (e.g., [26,12]) have been adopted to increase the decomposition efficiency.

Corresponding to the frequency-domain counterpart, the time series (AR/MA/ARMA) models with time-varying coefficients are also developed in simulations (e.g., [7]). The time series approach is efficient; however, it is difficult to determine the order and coefficients of the model. Apart from SRM and time series model, which can be regarded to be exact, several approximate alternatives associated with the orthogonal function representation were proposed in nonstationary or non-Gaussian process

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simulation, such as random trigonometric polynomials [8] and proper orthogonal decomposition (POD) (e.g., [14,19,15]). Although the more energy can be concentrated to a few modes through the orthogonal function decomposition, these approximate methods have some limitations. Firstly, the determination of the random coefficients is time-consuming. In addition, the convergence rate may not be fast enough. Furthermore, both of them require the covariance function as the simulation target. These functions are usually not available [18] and must be transformed from the prescribed spectra.

Similar to the challenges encountered in nonstationary process simulation, the methodology of EPSD estimation is not well established, especially as few samples, usually one sample, are available. Along with introducing the concept of evolutionary spectra, Priestley [20] proposed the classic EPSD estimation method based on the fixed window. Conte and Peng [3] developed the short-time Thomson's multiple-window approach for the scalar nonstationary process. Due to the better time-frequency resolution, wavelet transform-based approach [22,11] has been used to estimate evolutionary spectra for both scalar and vector-valued processes. In this method, the spectra estimation is based on the triangular relationship among the random process, wavelet coefficients and evolutionary spectra. By recasting the evolutionary spectra onto a series of wavelet bases (more accurately, the squared modulus of Fourier transform of wavelet function), the corresponding time coefficients can be estimated from the wavelet coefficients, which are derived from the multiple samples of the evolutionary nonstationary random process. The wavelets-based spectra estimation approach is also adopted to generate multiple nonstationary wind speed samples for the time-domain wind-induced structural response analysis [13].

In the present research, the proceeding relationships among the random process, wavelet coefficients and evolutionary spectra will be adopted for the nonstationary process simulation. The essence of the simulation idea is to approximately decouple the nonstationary process into a series of time modulated processes with the help of wavelets and these time modulated processes can be simulated taking advantage of FFT. Thus, the simulation speed can be greatly accelerated. The proposed simulation approach involves firstly approximating the prescribed time-dependent spectra of the nonstationary process as the summation of a series of the products of the squared modulus of Fourier transform of wavelets on different scales and corresponding time coefficient functions. Secondly, the squared modulus of Fourier transform of wavelets at different scales are regarded as independent processes and FFT-assisted SRM can be applied to each component in the summation. Lastly, the simulated time history from each component will be multiplied by the decomposed time coefficients, which are obtained from the proceeding time coefficient matrices via Cholesky decomposition. By summing up these products, the time history of the vector-valued nonstationary process to be simulated can be determined.

In this paper, a hybrid efficient simulation method is developed for the vector-valued nonstationary process, which includes spectra decomposition via wavelets and spectral representation method. Then the effectiveness and limitation of the proposed method will be studied using numerical examples. Furthermore, the validity of POD in nonstationary simulation by decomposing time-dependent spectra is discussed. Finally, some concluding remarks will be given.

2. Proposed simulation approach

Suppose that a vector-valued zero-mean nonstationary process with n components $\boldsymbol{x}(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^T$ admits ensuing instantaneous PSD matrix

$$\mathbf{S}(\omega,t)=\begin{bmatrix} S_{11}(\omega,t) & S_{12}(\omega,t) & \cdots & S_{1n}(\omega,t)\\ S_{21}(\omega,t) & S_{22}(\omega,t) & \cdots & S_{2n}(\omega,t)\\ \vdots & \vdots & \ddots & \vdots\\ S_{n1}(\omega,t) & S_{n2}(\omega,t) & \cdots & S_{nn}(\omega,t)\end{bmatrix} \tag{1}$$

where ω is circular frequency; $S_{jj}(\omega,t)$ is the auto nonstationary PSD of $x_j(t)$; $S_{jk}(\omega,t)$ is the cross nonstationary PSD between $x_j(t)$ and $x_k(t)$. The cross-correlation function between $x_j(t)$ and $x_k(t)$ is given by

$$\mathrm{R}_{jk}(t,t+\tau)=\int_{-\infty}^{\infty}\sqrt{S_{jj}(\omega,t)S_{kk}(\omega,t+\tau)}\gamma_{jk}(\omega)e^{i\omega\tau}\,d\omega \tag{2}$$

where $\gamma_{jk}(\omega)$ is the coherence function between $x_j(t)$ and $x_k(t)$. As $j=k$, the preceding equation reduces to the auto correlation function for the component process $x_j(t)$.

In the following, the proposed simulation approach for nonstationary processes with real and complex coherence functions will be presented separately and the reason for such a categorization will be discussed later.

2.1. Process with real coherence

The nonstationary spectra could be approximately expressed by the linear summation of products of the squared modulus of Fourier transform of wavelet function at different scales and associated time coefficients, i.e.,

$$S_{jk}(\omega,b)=\sum_{m=m_1}^{m_2}c_m^{jk}(b)|\widehat{\Psi}(\omega a_m)|^2 \tag{3}$$

where $\widehat{\Psi}(\omega)$ is the Fourier transform of wavelet function $\psi(t)$ defined as

$$\widehat{\Psi}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(t)e^{-i\omega t}\,dt \tag{4}$$

where a_m is the scale parameter; b is the time parameter; $m=m_1$, $m_1+1,\ldots,m_2$, m is the integer; m_1 and m_2 are numbers corresponding to lowest and highest scale levels. In practical application, the scale parameter is discretized by setting $a=a_m=\sigma^m$, where σ is positive constant.

The time-dependent coefficients $c_m^{jk}(b)$ can be determined by the following linear simultaneous equations:

$$\begin{bmatrix} Q_{m_1,m_1} & Q_{m_1,m_1+1} & \cdots & Q_{m_1,m_2}\\ Q_{m_1+1,m_1} & Q_{m_1+1,m_1+1} & \cdots & Q_{m_1+1,m_2}\\ \vdots & \vdots & \ddots & \vdots\\ Q_{m_2,m_1} & Q_{m_2,m_1+1} & \cdots & Q_{m_2,m_2}\end{bmatrix}\begin{bmatrix} c_{m_1}^{jk}(b)\\ c_{m_1+1}^{jk}(b)\\ \vdots\\ c_{m_2}^{jk}(b)\end{bmatrix}=\begin{bmatrix} P_{jk}(a_{m_1},b)\\ P_{jk}(a_{m_1+1},b)\\ \vdots\\ P_{jk}(a_{m_2},b)\end{bmatrix} \tag{5}$$

where

$$Q_{r,s}=2\pi a_r\int_{-\infty}^{\infty}|\widehat{\Psi}(\omega a_r)|^2|\widehat{\Psi}(\omega a_s)|^2d\omega\ (r,s=m_1,m_1+1,\ldots,m_2) \tag{6}$$

$$P_{jk}(a_r,b)=2\pi a_r\int_{-\infty}^{\infty}|\widehat{\Psi}(\omega a_r)|^2S_{jk}(\omega,b)\,d\omega \tag{7}$$

As $j=k$, the proceeding formulations will reduce to approximate auto spectra.

In the prior formulation, the wavelet function can be in arbitrary form, orthogonal or nonorthogonal, and real or complex for approximating the auto and cross spectra. The approximation scheme is very efficient as the system matrix is a non-singular Toeplitz matrix and also time-independent [22].

Previously, the similar formulations have been applied in evolutionary spectra estimation either for scalar processes [22] or vector-valued processes [11]. In these studies, the quantity $P_{jk}(a,b)$ is referred to as the mean value of instantaneous inner