Probabilistic Engineering Mechanics 37 (2014) 148–153

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Identification of composite uncertain material parameters from experimental modal data

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ARTICLE INFO

Article history:
Received 14 February 2013
Received in revised form
28 June 2014
Accepted 30 June 2014
Available online 18 July 2014

Keywords:
Composite structures
Uncertain parameter identification
Polynomial chaos
Pearson model
Experimental modal analysis

ABSTRACT

Stochastic analysis of structures using probability methods requires the statistical knowledge of uncertain material parameters. This is often quite easier to identify these statistics indirectly from structure response by solving an inverse stochastic problem. In this paper, a robust and efficient inverse stochastic method based on the non-sampling generalized polynomial chaos method is presented for identifying uncertain elastic parameters from experimental modal data. A data set on natural frequencies is collected from experimental modal analysis for sample orthotropic plates. The Pearson model is used to identify the distribution functions of the measured natural frequencies. This realization is then employed to construct the random orthogonal basis for each vibration mode. The uncertain parameters are represented by polynomial chaos expansions with unknown coefficients and the same random orthogonal basis as the vibration modes. The coefficients are identified via a stochastic inverse problem. The results show good agreement with experimental data.

1. Introduction

The behavior investigation of composite structural systems in which parameters are variable or not perfectly known is still a challenging problem. The most available models to investigate the behavior of composite structures assume an effective homogenized set of material properties. These models fail to capture the true behavior of the wide variety of such structures exhibit significant inherent uncertainty in material parameters. In this context, we face three critical issues. First, the modeling of such structures requires a large number of parameters. Secondly, direct experimental determination of these parameters by standard methods demands various test setup which are destructive and expensive. Finally, parameters of such structures exhibit uncertainty due to the structural complexity of individual components, interaction between the components, manufacturing process, etc. While deterministic models lead to nominal results, stochastic models, in contrast, capture these uncertainties and have become an important tool for analysis and design of composite structures. However, setting an appropriate stochastic model requires accurate assigning of parameter uncertainties which are often difficult to measure directly. These uncertainties will appear effectively in structure responses from which an inverse stochastic problem can be employed to identify uncertain parameters.

The unknown random parameters in a stochastic inverse problem are estimated as probability measures. The major stochastic methods use parametric probabilistic approach which allows the uncertain parameters of the nominal model to be taken into account through the introduction of prior probability models [1]. Such an approach consists directly constructing the probability distributions of the random quantities. In such cases, distributions of system outputs are characterized by certain types of probability density function (PDF), e.g. normal PDF. The main issue is to knowing the prior knowledge on distribution, particularly, on the PDF type of uncertain parameters. A usual solution is to use parametric PDF and identification of the parameters by using maximum likelihood estimation. A PDF which its parameters minimize the likelihood function is the best to represent the data. The application of the method is, however, limited due to the fact that the results depend on the model used and it can be sensitive to the choice of starting values [2]. Bayesian inference techniques are the most used methods which provide a robust approach to taking into account the system variability and parameter uncertainties. The method includes all known information about parameter and system uncertainties into a prior distribution model which can be combined with the likelihood to formulate the posterior PDF. It has been successfully used in many problems [3–6]. However, extreme dependence of higher order statistics to the form of prior distribution, using deterministic input sensor measurements, and providing no information for PDF of measured quantities cause that this framework has not been applied to design problems of multiple source of uncertainties. To overcome

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http://dx.doi.org/10.1016/j.probengmech.2014.06.008

these issues, in recent years, a method based on the spectral stochastic formulation was proposed. The generalized Polynomial Chaos (gPC) expansion plays the main rule in this method. The propagation of parameter uncertainties is approximated by the gPC expansion with unknown coefficients. Orthogonal polynomial basis functions are constructed according to the identified PDF for the uncertain measured data and parameters. The application of the method in various areas has been demonstrated in the past [7–14].

In this paper we focus on the identification of non-Gaussian uncertain parameters from real experimental modal data. The parameters are approximated by the gPC expansions where the unknown coefficients are estimated via a stochastic inverse problem. Specifically, the experimental eigenfrequencies of nominally identical laminated composite plates are used to estimate the coefficients of gPC expansion of uncertain material properties, i.e. elasticity moduli and the shear modulus. To provide the prior information on the PDF type of experimental data, the Pearson system is used [15]. Since the PDF identification by the Pearson system originally depends on the third and fourth statistical moments of observations, the identified PDF type is unique. Knowing the type of PDF provides prior information on the random variables and orthogonal polynomials which are used to construct the gPC expansions of data. The gPC expansions of the uncertain parameters are constructed assuming the same random variables with unknown coefficients. In this way, identification of the probability measure of the uncertain parameter is shifted to estimation of the unknown deterministic coefficients.

This paper is organized as follows: in the next section we present an introduction on gPC method for discretization of uncertain parameters. The stochastic direct and inverse modal analysis is discussed in Sections 3 and 4 respectively followed by PDF identification from experimental data in Section 5. Section 6 presents the numerical results of the procedure. The conclusions are discussed in the last section of the paper.

2. Discretization of uncertain parameters

Numerical stochastic solution of structural problems requires discretization of random quantities, e.g. uncertain parameters and structural responses. This is particularly essential for performing stochastic FEM. As a basic principle, the stochastic space has to be discretized for the treatment of randomness in the physical system to be adapted to the implementation of the deterministic FEM model. Several discretization methods have been adopted to integrate stochastic quantities to FEM simulation, cf. [16] for details. In sampling based methods, a random quantity is given for each realization of the structure model. In non-sampling methods, the uncertain model parameters and structural responses are represented by spectral decomposition with unknown coefficients and orthogonal polynomial basis. A least-squares fit can be used to determine the coefficients of the expansion. They use commonly the generalize polynomial chaos (gPC) expansion [17]. The basic idea is to project the random variables of problem onto a stochastic space spanned by a set of complete orthogonal polynomials. Let $(\Omega, \mathcal{A}, P)$ be a probability space, in which Ω is a sample space, $\mathcal{A}$ is a σ-algebra on Ω, and P is a probability measure on Ω. The gPC expansion of any uncertain parameter $\mathcal{P}:\Omega \longrightarrow \mathbb{R}$ with finite variance, i.e. $\mathcal{P} \in L^2(\Omega)$, can be represented as [18–20]

$$\mathcal{P} = \sum_{i=0}^{\infty} p_i \Psi_i(\boldsymbol{\xi}) = \boldsymbol{p}^T \boldsymbol{\Psi} \tag{1}$$

The vector $\boldsymbol{\Psi}$ is a set of orthogonal polynomials of multidimensional standard random vector $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_n\}^T$ defined on particular sample random spaces, i.e. $\xi_i \in \Omega_i, i = 1, 2, \ldots, n$. A set of (ξ_i, Ψ_i) can be selected depending on the type of the uncertain parameter, cf. [19,21,22] for details. The random orthogonal polynomials possess orthogonality property with respect to the inner product on $L^2(\Omega)$, i.e.

$$\mathbb{E}[\Psi_i, \Psi_j] = \mathbb{E}[\Psi_i^2]\delta_{ij}, \tag{2}$$

in which δ_{ij} represents the Kronecker delta and $\mathbb{E}$ denotes the expectation value with respect to the probability space. This property can be used to calculate the truncated gPC coefficients by projecting onto the orthogonal basis:

$$p_i = \frac{1}{\langle \Psi_i^2 \rangle} \int_{\Omega} \langle \mathcal{P}, \Psi_k(\boldsymbol{\xi}) \rangle \rho(\boldsymbol{\xi})\, \mathrm{d}(\boldsymbol{\xi}) \tag{3}$$

where $\rho(\boldsymbol{\xi})$ is the joint PDF corresponding to the random space Ω. In this way, the estimation of uncertain random parameter $\mathcal{P}$ is shifted to the calculation of deterministic coefficients p_i. These coefficients completely characterize the identification of uncertain parameter $\mathcal{P}$. Clearly, the accuracy of this characterization is hardly depended on the number of these coefficients and selection of orthogonal basis [19,22].

3. Stochastic FEM of composite modal analysis

Assuming general linear elastic behavior, the relation between stress and strain vectors, σ and ε respectively, is given by the generalized Hooke's law, i.e.

$$\{\sigma\} = [C]\{\varepsilon\} \tag{4}$$

where $[C] \in \mathbb{R}^{6\times 6}$ is the elastic matrix which is usually characterized with a set of engineering constants like generalized Young's moduli, shear moduli and Poisson's ratios. For an orthotropic material, there are twelve engineering constants. For this material model, the in-plane stress–strain relation equation (4) is reduced to

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{5}$$

The reduced stiffness coefficients C_{ij} are related to the compliances as follows:

$$C_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}, \quad C_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}, \quad C_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}, \quad C_{66} = G_{12}. \tag{6}$$

The Kirchhoff assumptions imply that the elastic behavior of an orthotropic material can be fully described by four instead of nine independent engineering constants, i.e. E_{11}, E_{22}, G_{12} and ν_{12}. Accordingly, the element stiffness matrix $\boldsymbol{K}_e$ in a FEM formulation can be derived as

$$\boldsymbol{K}_e = \int_{V_k} \boldsymbol{B}_e^T \boldsymbol{C}_{e_k} \boldsymbol{B}_e \,\mathrm{d}V_k \tag{7}$$

In which V_k is the volume element and $\boldsymbol{B}$ is the displacement–strain matrix. The relationship between the modal data and the elastic parameters of structure can be defined by this matrix. The element mass matrix $\boldsymbol{M}_e$ is stated as

$$\boldsymbol{M}_e = \int_{V_k} \gamma_{e_k} \boldsymbol{H}_a^T \boldsymbol{H} \,\mathrm{d}V_k \tag{8}$$

where γ is the mass density and $\boldsymbol{H}$ is the interpolation matrix. Assembling the element matrices to building the global stiffness and mass matrices, $\boldsymbol{K}$ and $\boldsymbol{M}$ respectively, yields the FEM model of undamped modal analysis of the structure as

$$[-\lambda_i \boldsymbol{M} + \boldsymbol{K}]\boldsymbol{u}_i = \boldsymbol{0}, \quad i = 1, 2, \ldots, n. \tag{9}$$