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On three- and two-dimensional fiber distributed models of biological tissues

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ABSTRACT

We describe three-dimensional and planar models of hyperelastic fiber reinforced materials characterized by statistical distribution of the fiber orientation. Our models are based on a second order approximation of the strain energy density in terms of the fourth pseudo-invariant $\bar{I}_4$, typically employed in the description of fiber reinforced materials. For a particular choice of the strain energy density associated to the fiber reinforcement, it is possible to derive the explicit expression of the material and spatial stress tensors and of the stress covariance tensors. The mechanical behavior of the models is assessed through uniaxial, biaxial and shear tests.

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1. Introduction

The theoretical and numerical modeling of the mechanical behavior of soft biological tissues in physiological or pathological conditions has received particular attention in the last few decades. One of the reasons of the interest is the necessity to use numerical models in medicine and biology to predict the behavior of organs or biological ensembles. Another reason is the design and the improvement of diagnostic tools and instruments that interact with biological tissues. Another motivation is the invention and the development of new materials mimicking peculiar features observed in natural tissues. The definition of a numerical model involving bio-tissues requires, in addition to the accurate measurements of the geometry, the use of sophisticated material models that must be calibrated against ad hoc experimental data. Note that the parameters of a material model are individual-dependent and often cannot be transferred easily from one case to another. Therefore, in view of actual applications, it seems to be wise to pursue the definition of material models with a reduced number of material parameters but otherwise able to cover a large range of deformations. In this way the calibration of the parameters may be somehow facilitated. The search of accurate material models characterized by a few material parameters is one of the key aspects of the modern computational biomechanics, and the present study tries to give a contribution by including the natural randomness of soft biological tissues.

In many situations of practical interest the non-pathological behavior of biological tissues is described sufficiently well by hyperelastic models that have been conceived for rubber materials, e.g., neo-Hookean, Mooney–Rivlin, Yeoh and others. On the other hand, most biological tissues are characterized by anisotropy, necessarily because the organ must provide a general multi-direction confinement and, at the same time, it must resist to localized and strongly oriented actions. Mechanical anisotropy is achieved by means of complex and specialized architectures of cable-like fibrils and fibers, made of the most diffused protein in nature, i.e., the collagen. It follows that material models commonly employed for biological tissues account for different kinds of anisotropy induced by the collagen cables [10,18,5,13,8]. Moreover, biological tissues are characterized ineluctably by a spatial distribution of the collagen reinforcement whereas unique strong alignments of fibers are in contrast with the function of the organ. Examples of distributed reinforcing fibers are found in the microstructure of the cornea [4,15,16,2] and of the artery walls, and in other biological tissues [7,17]. In the recent literature numerous material models considering a distributed orientation of the collagen fibers have been presented and discussed, starting from the seminal work by Lanir [14], and including several important contributions [7,6,1]. In this regard, our recent work on this field [19] proposed a novel point of view by introducing the concept of second order (or variance) approximation of the strain energy density of a fiber distributed material in terms of the fourth pseudo-invariant $\bar{I}_4$. The model was developed to overcome the

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two main difficulties of non-deterministic mechanical properties: (i) the impossibility to obtain the analytical definition of stress and elasticity tensors and (ii) the heavy computational effort necessary to compute stress and elasticity tensors in numerical applications.

Statistically based material models are appealing in stochastic approaches when they are proved to perform well also in terms of covariance stress tensors. The discussion on the covariance stress tensor of the second order approximation model is one of the new contributions of the present paper on the line of [23].

In several tissues geometrically organized as structures, e.g., skin, corneas, irides, artery walls, and other shell-like or membrane like organs, the distribution of the collagen fibers assumes a prevailing two-dimensional configuration [2,20]. The attractive possibility of using membrane and shell theories to reduce the computational effort in view of evaluating the mechanical behavior of organs calls for the development of constitutive models characterized by planarity of the fiber micro-structural organization, see, e.g., [21,24]. As an additional new contribution of this work, we present here the two-dimensional version of the second order approximation material model above recalled [19].

The organization of the paper is as follows. In Section 2 we introduce briefly the hyperelastic framework and the necessary definitions. In Section 3 we recall the basic ideas of the three-dimensional material model presented in [19]. Under the assumption of an axis-symmetric distribution of the fiber orientation, we derive a particularly compact analytical expression of the second Piola–Kirchhoff stress and of the covariance stress tensor. In Section 4 we particularize the three-dimensional model to a specific plane, containing the distribution of fibers. In Section 5 we compare, through uniaxial, biaxial and shear test, the behavior of the two-dimensional and three-dimensional distributions, in terms of stress and covariance stress components. The behavior of the model as a function of concentration parameter of a von Mises distribution is also discussed.

2. Hyperelasticity framework

In the framework of nonlinear continuum mechanics, we postulate the existence of a Helmholtz free-energy density per unit reference volume Ψ. We comply with the purely elastic case, where the free energy is assumed to be dependent on the deformation gradient $\mathbf{F}$ only, i.e., $\Psi = \Psi(\mathbf{F})$. For a biological tissue with collagen fibers it is customary to decompose additively the strain energy into three terms:

$$\Psi = \Psi_{\text{vol}} + \Psi_{\text{iso}} + \Psi_{\text{aniso}}. \quad (1)$$

The first term, Ψ_{vol}, accounts for volume changes, and it is assumed to be dependent on the volumetric part of the deformation, i.e., on the Jacobian $J = \det \mathbf{F}$, i.e.,

$$\Psi_{\text{vol}} = \Psi_{\text{vol}}(J).$$

The second term, Ψ_{iso}, accounts for the isotropic behavior of the material due to the underlying matrix and eventually to a portion of isotropically distributed fibrous reinforcement. Usually, Ψ_{iso} is assumed to be dependent on the first and second invariants, $\bar{I}_1$ and $\bar{I}_2$, of the modified Cauchy–Green deformation tensor $\overline{\mathbf{C}} = \overline{\mathbf{F}}^T \overline{\mathbf{F}}$, where $\overline{\mathbf{F}} = J^{-1/3}\mathbf{F}$:

$$\Psi_{\text{iso}} = \Psi_{\text{iso}}(\bar{I}_1, \bar{I}_2).$$

The anisotropic effect of the fibrous reinforcement is described by the third term Ψ_{aniso}. According to a standard approach initiated by Spencer [22], here Ψ_{aniso} is assumed to be dependent on the modified tensor $\overline{\mathbf{C}}$ and on particular vectors – or tensors – describing the intrinsic microstructure of the material. As a consequence of the additive decomposition (1) and of the decoupling of the arguments between the addends, it follows that the second Piola–Kirchhoff stress tensor $\mathbf{S}$ splits into the sum of three terms:

$$\mathbf{S} = \mathbf{S}_{\text{vol}} + \mathbf{S}_{\text{iso}} + \mathbf{S}_{\text{aniso}},$$

in the form:

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}} = 2\frac{\partial \Psi_{\text{vol}}}{\partial \mathbf{C}} + (\overline{\mathbf{S}}_{\text{iso}} + \overline{\mathbf{S}}_{\text{aniso}})\frac{\partial \overline{\mathbf{C}}}{\partial \mathbf{C}},$$

where

$$\overline{\mathbf{S}}_{\text{iso}} = 2\frac{\partial \Psi_{\text{iso}}}{\partial \overline{\mathbf{C}}}, \quad \overline{\mathbf{S}}_{\text{aniso}} = 2\frac{\partial \Psi_{\text{aniso}}}{\partial \overline{\mathbf{C}}}. \quad (2)$$

Explicit expressions for the anisotropic second Piola–Kirchhoff stress can be found in standard continuum mechanics textbooks [9].

According to [10], in the case of a single family of parallel fibers oriented in the referential direction $\mathbf{a}_0$, a well accepted form of the anisotropic Helmholtz free energy density is given by

$$\Psi_{\text{aniso}}(\bar{I}_4) = \overline{\Psi}_{\text{aniso}}(\bar{I}_4) + \Psi^0_{\text{aniso}} = \frac{k_1}{2k_2}\exp[k_2(\bar{I}_4 - 1)^2] - \frac{k_1}{2k_2},$$

where the pseudo-invariant $\bar{I}_4$ is the contraction of $\overline{\mathbf{C}}$ and of the second order structure tensor $\mathbf{A}_0 = \mathbf{a}_0 \otimes \mathbf{a}_0$, i.e.,

$$\bar{I}_4(\mathbf{a}_0) = \overline{\mathbf{C}} : \mathbf{A}_0.$$

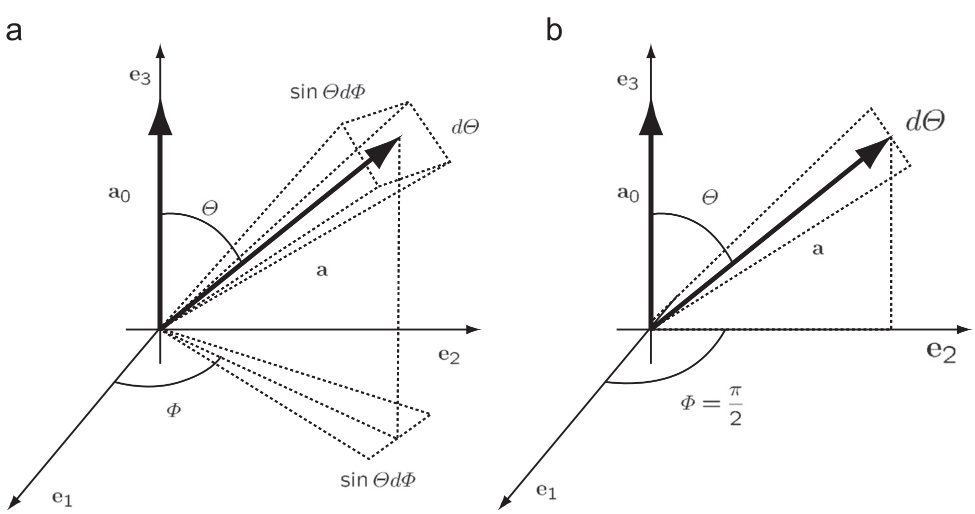

Fig. 1. Orientation of the generic unit vector **a** aligned with a portion of fibers. (a) Spherical coordinates for a fully three-dimensional distribution. (b) Cylindrical coordinates for a planar distribution.

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