Mechanism and Machine Theory 84 (2015) 25–36

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Determination of the maximal singularity-free workspace of 3-DOF parallel mechanisms with a constructive geometric approach

Mohammad-Hadi Farzaneh Kaloorazi [a], Mehdi Tale Masouleh [a,*], Stéphane Caro [b]

[a] Human-Robot Interaction Laboratory (TAAR-Lab), Faculty of New Sciences and Technologies, University of Tehran, Iran
[b] CNRS-IRCCyN, UMR 6597, 1 rue de la Noë, 44321 Nantes, France

ARTICLE INFO

Article history:
Received 31 May 2014
Received in revised form 13 October 2014
Accepted 14 October 2014
Available online 31 October 2014

Keywords:
Parallel mechanisms
Singularity-free workspace
Geometric approach
Offset curve algorithm

ABSTRACT

This paper proposes a novel approach to obtain the maximal singularity-free regions of planar parallel mechanisms, which is based on a constructive geometric reasoning. The proposed approach consists of two algorithms. First, the borders of the singularity-free region associated with an arbitrary starting point of the moving platform are obtained. Then, the second algorithm finds the center of the maximal singularity-free circle, which is obtained using the so-called offset curve algorithm. The procedure is applied to a 3-PRR planar parallel mechanism as an example and the obtained results illustrate graphically the effectiveness of the proposed algorithm. The proposed approach can be directly applied to obtain the maximal singularity-free circle of similar parallel mechanisms, which is not the case for other approaches proposed in the literature that are limited to a given parallel mechanism, namely, the 3-RPR planar parallel mechanism. Moreover, as the main feature of the proposed approach, it can be implemented both in a CAD system and in a computer algebra system where non-convex and reentrant curves can be considered.
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1. Introduction

Parallel mechanisms (PMs) are robotic mechanical systems composed of one moving platform and one base connected by at least two serial kinematic chains [1–3].

The last two decades have witnessed a noticeable rise in the number of publications regarding the kinematic and dynamic analyses of PMs to propose the most promising design. PMs have their own drawbacks and even a simple one can lead to a complicated kinematic analysis. In general, when a PM is not symmetrical, its geometry and kinematic analysis are usually complex.

The singularities of PMs should also be carefully studied as PMs may gain or lose some DOF, and consequently become uncontrollable, in such configurations [1].

These configurations can be mathematically related to the singularity of some Jacobian matrices arisen from the first-order kinematic properties of the mechanism [4]. Designing a PM with a singularity-free workspace is a vital condition for further analysis, such as path planning and control.

This paper aims at obtaining the Maximal Singularity-Free Circle (MSFC) of 3-DOF planar PMs for a given orientation of their mobile platform.

A workspace in the shape of circle is chosen for the sake of simplicity. However, the developed algorithm can be used to obtain a maximal singularity-free workspace of any other shape. It is worth noting that this study is a first step toward the dimensional synthesis of PMs for which a desired singularity-free workspace, i.e., MSFC, is prescribed.

* Corresponding author.
E-mail address: m.t.masouleh@ut.ac.ir (M.T. Masouleh).

http://dx.doi.org/10.1016/j.mechmachtheory.2014.10.003
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To the best of our knowledge, in the literature, results of the MSFC have been obtained only for a prescribed center point and this assumption bounds the radius of the circle and results into a local optimum solution. In this paper, the center point of the MSFC is not prescribed and is found by using a geometrical reasoning. It should be noted that the MSFC is readily computed once the center point is obtained. The proposed approach for obtaining the center point of the MSFC is based on a novel constructive geometric procedure, which is the main contribution of the paper.

The first proposed algorithm, called conceptual algorithm (*Alg. Conc*), presents a conceptual approach in order to obtain the singularity-free region of PMs, which could be applied to non-convex singularity locus. Afterwards, practical algorithm (*Alg. Prac*) presents a practical method based on *Alg. Conc*. Moreover, an offset curve algorithm, *Alg. Offs*, is adapted for the geometric purpose of this work. Offset curve algorithms [5,6] are geometric constructive tools, which have diverse engineering applications and have consequently motivated extensive researches concerning various offset techniques. They play an important role in numerical control and CAD/CAM applications [5]. To the best of our knowledge, the problem of MSFC has never been investigated upon a geometric standpoint. The proposed algorithm, which is inspired from geometric properties associated to the MSFC, could be implemented either in a computer algebra system or using a CAD system.

Through this paper, in order to illustrate the proposed approach, as a case study, the procedure of obtaining the MSFC is applied to a 3-PRR planar PM. Note that, P stands for an actuated prismatic joint and R stands for a passive revolute joint. However, it can be extended to all planar 3-DOF PMs presented in [7]. To the best of our knowledge, 3-RPR and 6-UPS (SPS) PMs have been widely treated in the literature since they lead respectively to quadratic and cubic polynomial expressions for their singularity locus which simplifies considerably the mathematical challenge. There has been an extensive study conducted on the singularity-free workspace of PMs where most of them are based on complicated numerical approaches and entail some limits.

Bonev et al. [8] conducted an exhaustive study on the singularity locus of planar 3-DOF PMs by resorting to screw theory. In [9], a method based on the geometrical parameters is proposed for which the singularity-free workspace of a three-legged PM is obtained. The search for the maximum singularity-free circle of 3-DOF PMs can be expressed mathematically as an optimization problem accompanied with a constraint resorted to the Lagrangian multipliers [10].

Jiang and Gosselin [11–14] proposed some numerical techniques to find the singularity-free workspace of 3-DOF PMs. Recently, in [15], upon resorting to particle swarm optimization the maximum singularity-free circle of a 3-DOF PM was obtained for a prescribed center point. In [16], Mousavi et al. obtained the maximal singularity-free ellipse included in the workspace of a 6-UPS PPM, using convex optimization. Moreover, in [17], the problem of closeness to singularity is addressed by formulating the question in terms of a constrained optimization problem. In [18], an interval-based method is introduced to obtain the maximal singularity-free sphere, in the constant-orientation workspace of parallel robots. The approach is applicable for almost all parallel robots, but in the case of high degree polynomial of singularity, the procedure leads to a very time consuming computation.

A minor modification in the kinematic arrangement, for instance having a 3-PRR PM instead of 3-RPR PM, leads to the complexity of the procedure for which methods reported in [19–21,11,10,9,7] are not applicable and fail to provide satisfactory results. One of the problems in such investigations is the presence of the square roots in the singularity loci expressions. The proposed algorithm is split into two parts: (1) the first part deals with the algorithm used to obtain the subregion of interest for the MSFC, which is presented in two forms; *Alg. Conc* in concept and *Alg. Prac* in practice, and (2) a second algorithm is developed to obtain the center point of the MSFC for the foregoing subregion obtained from the first part, called *Alg. Offs*.

The remainder of this paper is organized as follows. First, the kinematic properties of the PM under study, i.e., the 3-PRR PM, are broadly reviewed. The two proposed algorithms, *Alg. Conc* and *Alg. Prac*, are presented and fully described to the end of obtaining the singularity-free region. Finally, the offset curve algorithm, called *Alg. Offs*, is introduced and used in order to obtain the center and radius of the MSFC form singularity-free region.

2. Kinematic review of a 3-PRR planar parallel mechanism

A 3-PRR planar PM, Fig. 1(a), consists of three kinematically identical limbs actuated by a prismatic joint fixed at the base and followed by two passive R joints, as depicted in Fig. 2. As it can be observed from Fig. 2, O_{xyz}, with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ as unit vectors, represents the fixed frame and $O'_{x'y'z'}$ stands for the moving frame. The pose (position and orientation) of the moving-platform is defined by (x, y, ϕ) where $\mathbf{p} = [x, y]^T$ and ϕ represents respectively the Cartesian position and the orientation of the moving frame with respect to the fixed frame. Upon resorting to *screw theory* [7], the kinematic Jacobian matrix $\mathbf{J}$ of the mechanism can be formulated as follows:

$$\mathbf{J} = \left[\begin{array}{cc} \mathbf{l}_1 & \mathbf{r}_1 \times \mathbf{l}_1 \\ \mathbf{l}_2 & \mathbf{r}_2 \times \mathbf{l}_2 \\ \mathbf{l}_3 & \mathbf{r}_3 \times \mathbf{l}_3 \\ \hline \mathbf{0} & \hat{\mathbf{i}} \\ \mathbf{0} & \hat{\mathbf{j}} \\ \hat{\mathbf{k}} & \mathbf{0} \end{array}\right], \tag{1}$$

in which $\mathbf{l}_i$, $i = 1, 2, 3$, is the unit vector of the line connecting point B_i to point C_i and $\mathbf{r}_i$ is the vector connecting the origin of the moving platform to point C_i. Singular configurations of the mechanism occur when the Jacobian matrix $\mathbf{J}$ becomes rank deficient [22], i.e., the determinant of the foregoing matrix vanishes, $det(\mathbf{J}) = 0$. The latter condition leads to a polynomial of degree 20 (20 in y and 16 in x) for a constant-orientation of the moving platform [7]. It is worth noticing that the latter polynomial corresponds to the eight working modes of the mechanism and, as reported in [7], it is not possible to find a polynomial expression for a single working mode among the

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