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Non-dimensional approach for static balancing of rotational flexures

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ABSTRACT

This work presents a general method of statically balancing flexural hinges. Using a set of non-dimensional parameters, we show that one can quickly design a statically balanced rotational flexure. First, a balancing method is developed for an idealized hinge/torsion spring system. This method is then extended to load-dependent systems and is demonstrated with the design of a balanced cross-axis-flexural pivot with stiffness that varies as a function of compressive preload. A physical prototype is built and tested to verify the design method. This method is general for any system that has a well understood stiffness response to an applied compressive load.

1. Introduction

The objective of this research is to develop a general method for the approximate static balancing of compliant hinges. A compliant mechanism obtains its motion from the deflection of its constituent members. Because this eliminates sliding contact of surfaces, friction and subsequent wear can be avoided, leading to higher performance [1]. Because of the strain energy associated with bending the flexible members, compliant mechanisms generally have higher actuation effort compared to traditional mechanisms [2]. This can require larger actuators, which increases mass and cost. Static balancing is one strategy for reducing the actuation effort of compliant mechanisms [3–8].

Static balancing is often accomplished by adding auxiliary springs that provide energy storage [3,4]. As the mechanism is actuated, energy stored in the balancing elements is transfered to the deflected mechanism [6]. This means that less energy must be added during actuation, thereby reducing actuation effort [5]. This strategy has been effectively incorporated into applications such as the design of surgical instruments and prosthetics [6–8].

Balancing elements commonly incorporate a negative stiffness mechanism, such as buckled beams in linear systems [9], or preloaded linear springs in rotational systems [10]. Other approaches use gravity balancing or systems of ideal springs [6,11,12].

Static balancing strategies do exist that do not require optimization; these rely on mathematically exact solutions [13,14]. However, the design of statically balanced systems often requires the use of optimization routines [9,15,16]. Usually, the optimization problem minimizes the change in a mechanism's stored energy or searches for an appropriate negative-stiffness mechanism [10]. Depending on the system under consideration, this optimization may incorporate finite element analysis (FEA) and topology optimization. This means that to design a statically balanced compliant mechanism, significant resources must be available to develop and validate the model being used. Additionally, optimization routines utilizing FEA can quickly become cumbersome due to the relatively long solution time of non-linear FEA and the many function calls of most optimization routines.

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Finally, building practical statically balanced mechanisms is difficult because the balancing element is often bulky, making the system much larger than what is convenient [12].

The method herein presented provides an approximate solution to the static balancing problem. Although perfect balancing is not achieved, an average reduction in stiffness of 87% is achieved for an eighty degree range of motion. This method does not require FEA and can avoid some of the optimization required by other approaches for static balancing.

2. Nomenclature

In this work, "load-independent (LI) joint" is a joint with a rotational stiffness that is not a function of applied lateral loads. This is modeled as a pin joint with a torsional spring. A "load-dependent (LD) joint" is a joint whose stiffness changes when a lateral load is applied. An LD joint can be modeled as an LI joint if a relationship can be found between the applied lateral loads and the joint stiffness.

In this work the statically balanced system consists of an LI joint of finite, constant stiffness that is balanced by the addition of a preloaded constant-stiffness linear spring. The spring connects at points equidistant from the pivot, as shown in Fig. 1. This simplified system can represent load-dependent systems with proper application of the pseudo-rigid-body model [1].

Variables and their relationships are included in the following lists. The first list is for variables directly related to balancing of LI compliant hinges, as illustrated in Fig. 1.

k_θ	Torsional stiffness of the LI joint or corrected stiffness of the LD joint (with applied loads)
k_l	Stiffness of balancing spring
k	Stiffness of balanced system
d	Distance from pivot center to balancing spring attachment points
x_0	Free length of balancing spring
$P = k_l(2d - x_0)$	Preload applied to balancing spring
θ	Angle of deflection of the LI or the LD joint
$T = k\theta$	Torque required to deflect hinge through angle θ
$\Pi_1 = k_\theta/(Pd)$	Pi group governing torsional stiffness
$\Pi_2 = k_l d/P$	Pi group governing stiffness of balancing spring

The following list contains variables related to the design of a cross-axis-flexural pivot (CAFP) that has stiffness that is load-dependent. See Fig. 2 for a depiction of geometric variables.

E	Young's modulus of the flexure material
b	Width of CAFP flexure strip
t	Thickness of CAFP flexure strip
$I = bt^3/12$	Moment of inertia of CAFP flexure strip
L	Length of CAFP flexure strips
k'_θ	Uncorrected torsional stiffness of the LD joint (no applied loads)

The following list contains variables used to correct the stiffness of a CAFP to account for the effects of applied loads (adapted from Wittrick [17]). See Fig. 2 for a depiction of geometric variables. The loads V and H are applied to the moving block of the CAFP.

V	Vertical load applied to hinge
H	Horizontal load applied to hinge
α	Half the intersection angle of the CAFP flexures
$v = VL^2 \sec(\alpha)/(EI)$	Non-dimensionalized applied vertical load
$h = HL^2 \csc(\alpha)/(EI)$	Non-dimensionalized applied horizontal load

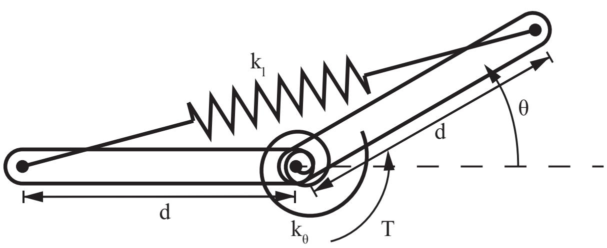

Fig. 1. An LI system with balancing spring and associated variables.