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Fuzzy-small degrees of freedom representation of linear and angular variations in mechanical assemblies for tolerance analysis and allocation

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ABSTRACT

Tolerances naturally generate an uncertain environment for design and manufacturing. In this paper, a novel fuzzy based tolerance representation approach for modeling the variations of geometric features due to dimensional tolerances is presented. The two concepts of fuzzy theory and small degrees of freedom are combined to introduce the fuzzy-small degrees of freedom model (F-SDOF). This model is suitable for tolerance analysis of mechanical assemblies with linear and angular tolerances. Based on the fuzzy concept, a new index (called the assemblability index) is introduced which signifies the fitting quality of parts in the assembly. Graphical and numerical representations of tolerance allocation by this method are presented. The goal of tolerance allocation is to adjust the tolerances assigned at the design stage so as to meet a functional requirement at the assembly stage. The presented method is compatible with the current dimensioning and tolerancing standards. The application of the proposed methodology is illustrated through presenting an example problem.

1. Introduction

Mechanical products are usually made by assembling several parts. The dimensional and geometric variations of each part have to be specified by tolerances which guarantee a certain level of quality in terms of satisfying functional requirements. Tighter tolerances provide better performance but increase the manufacturing cost considerably. Therefore, there is a great desire to apply tolerance analysis to predict the accumulation effects of the allocated component tolerances on the functional requirements of the assembly. However, the success of tolerance analysis is conditional upon the adopted mathematical model that describes the variations of the assembly function due to component tolerances.

Tolerances in a mechanical assembly can be divided in two categories; manufacturing tolerances (part tolerances) and assembly tolerances (accumulation tolerances). The assembly tolerances are the allowances on the design requirements. The assembly function is the most essential equation for tolerance analysis and allocation that describes relations between the assembly and manufacturing tolerances. Various tolerance modeling approaches have been presented in the literature. A survey of mathematical methods for tolerance modeling is presented by Pasupathy et al. [1]. Chase et al. [2] presented the direct linearization method (DLM) for tolerance analysis of the assembly using small kinematic adjustment of component dimensions based on the linear approximation of implicit dimensional constraint functions. Wittwer et al. [3] applied the direct linearization method to position error in mechanisms due to link-length and angle variations. The TTRS model was initially introduced by Desrochers et al. [4] to model tolerance zones. A tolerance zone related to a TTRS is described by a displacement torsor which is a six component vector that includes three rotations and three translations of the feature within its tolerance zone in 3D space [4]. Desrochers [5] presented a mathematical modeling of three dimensional tolerance zones based on torsor representation. The

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Tolerance-Mapping (T-Map) model was proposed by Davidson et al. [6]. A T-Map is a set of points resulting from a one-to-one mapping from all the variational possibilities of a feature, within its tolerance-zone, to the Euclidean pint-space [6]. Huang and Zhang [7] developed a robust tolerance design method to quantify the effects of uncertain factors on the accuracy of function generation mechanisms with joint clearances. Bruye`re et al. [8] presented a digital simulation based on vectorial dimensioning, tooth contact analysis and Monte Carlo simulation for tolerance analysis of gears. Dantan et al. [9] proposed a statistical method for tolerance analysis and used genetic algorithm for tolerance synthesis. Wang [10] presented a new semantic tolerance modeling scheme based on modal interval to develop the interpretability of tolerance modeling. Wu and Rao [11] proposed a fuzzy based approach using vector loops for modeling and analysis of mechanical errors caused by various geometric and dimensional tolerances in a mechanical assembly. Movahhedy and Khodaygan [12] modified worst case (WC) and root-sum-square (RSS) methods for tolerance analysis of mechanical assemblies with asymmetric and unilateral tolerances. Khodaygan and Movahhedy [13] presented a new method for tolerance analysis of mechanical assemblies with asymmetric tolerances based on uncertainty–accumulation models. According to this scheme, toleranced components were described as fuzzy numbers with their membership functions constructed using the statistical distributions of manufactured variables [13]. Khodaygan et al. [14] recently introduced a feature based approach for tolerance analysis of the mechanical assemblies established upon modal interval mathematics and small degrees of freedom (MI-SDOF) concepts.

In this paper, a novel method for modeling linear and angular variations in the mechanical assemblies is presented based on the fuzzy theory and small degrees of freedom concepts. The uncertainty in the dimensions and geometrical form of features due to the tolerances are mathematically described by fuzzy numbers. By combining the fuzzy theory and small degrees of freedom (called F-SDOF for short) concepts, a proper methodology for tolerance analysis is proposed.

This paper is organized as follows. In Section 2, the concepts involved as the basis of the proposed method are described. In Section 3, a new method for dimensional tolerance modeling based on combined fuzzy-SDOF concept is developed. In Section 4, the formulation of the F-SDOF method for estimation of the accumulation of dimensional tolerances is presented. In Section 5, the tolerance allocation procedure based on the assemblability index is presented. Section 6 presents the proposed tolerance analysis and allocation algorithm based on the F-SDOF model. Finally, an example application is presented in Section 7 followed by the conclusions in Section 8.

2. Basic concepts

In this section some basic concepts necessary for the proposed model are presented.

2.1. Fuzzy logic and alpha-cut method

Fuzzy logic has been introduced as an extension of the classical view of set. The fuzzy set theory permits the gradual assessment of the membership of elements in a set that is described with a membership function valued in the interval [0, 1]. Fuzzy system modeling has been studied to deal with complex, not clearly explained and uncertain systems. Since part dimensions are inherently prone to uncertainty due to manufacturing variations, fuzzy numbers are an ideal medium to represent such uncertainty. Fuzzy numbers are particularly useful in modeling functional tolerances where the allowable variation is typically subjective. Uncertain parameters are considered to be fuzzy numbers with some membership functions [15]. The fuzzy set that contains all elements with a membership of $\alpha\in[0,1]$ and above is called *α-cut* of the membership function. *α-cut* of a fuzzy set as A is

$$A_\alpha = \{x|\mu_A > \alpha\}, \alpha \in [0,1]. \quad (1)$$

The alpha-cut method is used to represent the uncertainty or imprecision in the variable(s) [17].

2.2. Modal interval arithmetic

In many applications, the data for analysis contain a certain degree of uncertainty (e.g. measures of dimensions). Such data is naturally in interval form. Interval mathematics is a tool that is devised to deal with intervaled data [16]. In the interval mathematics, an interval is a closed non-empty set of real numbers, where x_L is the lower bound, and x_U is the upper bound. This set is a real interval and is denoted by $X=[x_L,x_U]$. Gardenes et al. [17] presented the modal interval arithmetic as a logical extension of the traditional interval arithmetic. Unlike the interval arithmetic that identifies an interval by a set of real numbers, the modal interval arithmetic identifies an interval by a set of predicates which is satisfied by the real numbers. For example in the modal interval arithmetic both $[x,y]$ and $[y,x]$ are valid intervals but according to traditional interval arithmetic they are not. The modal interval arithmetic operations on intervals are presented in the Refs. [14,18].

2.3. Small degrees of freedom concept

Geometrical and dimensional tolerances can be described by tolerance zones, such that the toleranced feature is able to rotate and translate within its tolerance zone. Let $\boldsymbol{\varepsilon}_{DOF}$ be the small translation vector and $\boldsymbol{\omega}_{DOF}$ the small rotation vector that are defined on a tolerance feature. u, v and w are components of small translations along x, y and z axes, respectively, and α, β and γ are components of