Mechanism and Machine Theory 76 (2014) 1–19

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

A novel method based on mechanical analysis for the stretch of the neutral line of the flexspline cup of a harmonic drive

Xiaoxia Chen [a], Yusheng Liu [b,*], Jingzhong Xing [a], Shuzhong Lin [a], Ming Ma [c]

[a] *Tianjin City Key Lab of Modern Mechatronics Equipment Technology, Tianjin Polytechnic University, Tianjin 300387, China*
[b] *The State Key Lab of CAD&CG, Zhejiang University, Hangzhou, 310027, China*
[c] *School of Textiles, Tianjin Polytechnic University, Tianjin 300387, China*

ARTICLE INFO

Article history:
Received 30 January 2013
received in revised form 20 January 2014
accepted 24 January 2014
Available online 16 March 2014

Keywords:
Theoretical model
Harmonic drive
Mechanical analysis
Stretch

ABSTRACT

The traditional harmonic drive (HD) model is often simplified by assuming that the neutral line in a flexspline (FS) tooth rim is inextensible. The circumferential strain induced by the circumferential force is also always neglected. These simplifications may induce interference between the teeth and excessive deformation or may even require clearance to overcome the jamming resulting from the interference. In this study, a novel method based on mechanical analysis is developed for the stretch of the neutral line of the FS of an HD. First, the internal forces and the deformation of the neutral line of FS are formulated as a set of differential equations with a geometric constraint and equilibrium equations with continuous conditions. Next, the circumferential strain is calculated using Hooke's law for the circumferential force in the assembly state. The stretch between two adjacent teeth of the tooth rim is calculated by solving an integral equation for the circumferential strain. Finally, FEA models are constructed to verify the theoretical results and to determine the circumferential strains in the tooth rim on the FS. The simulation results demonstrate that the method developed is highly accurate.

1. Introduction

A harmonic drive (HD) [1] is a special gear-drive speed reduction system that operates on the principle of periodic elastic deformation of a flexspline (FS) instead of the rigid-body motion of a general gearing system. A HD has three components: a wave generator (WG), a flexspline, and a circular spline (CS). Many studies have been conducted on HD tooth profiles, kinematics, deformation, stress, position accuracy and dynamic behavior [2–5] to improve HD performance. HDs yield a high transmission ratio in extremely compact forms and are therefore widely used in many industrial fields, such as robotics, military applications, aerospace applications, nuclear energy, precision positioning systems, and semiconductor manufacturing.

Since the HD was invented, many methods have been developed for the modeling, analysis, and design of HDs. For technical reasons, involute tooth profiles have often been used instead of truly conjugate tooth profiles. Conventional involute profiles may lead to tip interference, which has resulted in the use of modified involute profiles. Ivanov [6] and Shen & Ye [7] used an approximate method to investigate the performance of tooth profiles. To maximize the HD fatigue life, Oguz & Fehmi [8] performed a shape optimization of the involute tooth profile using a finite element method (FEM). A double-circular-arc tooth profile was developed to improve the torsional stiffness and transmission accuracy of a HD [9–11]. Maiti & Roy [12] consider HD as a special type of two gear epicyclic drive, and point out with two tooth difference, tooth tip interference can avoided in HD. Maiti [13] introduced a new WG with uncorrected involute profiles in which the WG was modeled as a cam of circular and elliptical arcs. Kondo & Takada [14] developed a new method to determine the conjugate profiles in which the common tooth normal to the meshing point passed through the pitch point. In addition, Dong et.al [15,16] developed a rigorous model to describe the FS motion and the HD tooth meshing to establish the fundamental

* Corresponding author. Tel.: +86 88206681 524; fax: +86 88206680.
E-mail address: ysliu@cad.zju.edu.cn (Y. Liu).

0094-114X/$ – see front matter
http://dx.doi.org/10.1016/j.mechmachtheory.2014.01.014

Nomenclature

r_m	Radius of the curve of the neutral line before deformation
u	Radial displacement of the neutral line after deformation
ρ	Radial vector of the neutral line after deformation
u_0	Maximum radial deformation of the neutral line
D	Calculated radius of the roller
β	Polar angle between the contact point of the roller and the maximum radius
X_1	Bending moment of the horizontal section
X_2	Circumferential force on the horizontal section
X_3	Shear force on the horizontal section
M_1, M_2	Bending moment in any section
F_{N1}, F_{N2}	Circumferential force in any section
ε_H	Circumferential strain
EA	Equivalent stretch stiffness of the tooth rim
b, h	Width and height of the rectangular cross section of the tooth rim
I_z	Moment of inertia of the cross section of the tooth rim
Δs_i	Stretch between two adjacent teeth of the tooth rim

kinematical principle of HD operation. A method was also developed to describe the conning effect deformation of the neutral layer under loading [17]. When engagement occurs in the tooth tip of pairs, the deformation of the tooth tip becomes significant in the engagement analysis of the HD. To investigate the deformation of the tooth tip and the neutral layer both in the assembly state and the transmission state, a shell element was used to model the FS cone and the teeth on the FS [18,19]. In order to improve the engagement properties to provide a longer fatigue life, Ishikawa [20] introduced a point-symmetrical curve as a tooth profile of HD. However, the precise cutting of the tooth profile is not easy. The tooth profile is approximated by two circular arcs by taking the conning effect deformation of FS into consideration [9].

All traditional HD studies have assumed an inextensible neutral line for the FS deformation, which greatly simplifies the deformation and engagement analysis. The simplified geometric relationship between the radial and circumferential displacements leads to a set of linear geometric relationships and equilibrium equations both for the geometrical and mechanical analysis. However, the stretch in the neutral line in the assembly state calculated by the geometrical method [21] and the stretch in the assembly and transmission states calculated by the finite element analysis (FEA) model [22] both clearly show tension or compression in the neutral line; therefore, the inextensible neutral line assumption needs to be relaxed.

In this study, an efficient mechanical analysis is developed to accurately calculate the stretch in the neutral line of the tooth rim and the effects of the stretch on the tooth root position. First, the internal forces and deformation are calculated for the neutral line of the tooth rim by solving a set of differential equations with a geometric constraint and equilibrium equations with continuous conditions. Then, the circumferential strain is obtained using Hooke's law for the circumferential force, and the stretch between two adjacent teeth of the tooth rim is obtained by solving an integral equation for the circumferential strain. To verify the theoretical results, three FEA models are built in ANSYS to determine the circumferential strains in the FS tooth rim.

This paper is organized as follows. The theoretical calculation of the internal forces and the displacement of the tooth rim are discussed in Section 2. The internal forces and the deformation of the tooth rim for a given maximum radial FS displacement is formulated in Section 3. The effect of the roller location angle β on the internal forces and deformation is discussed in Section 4. The

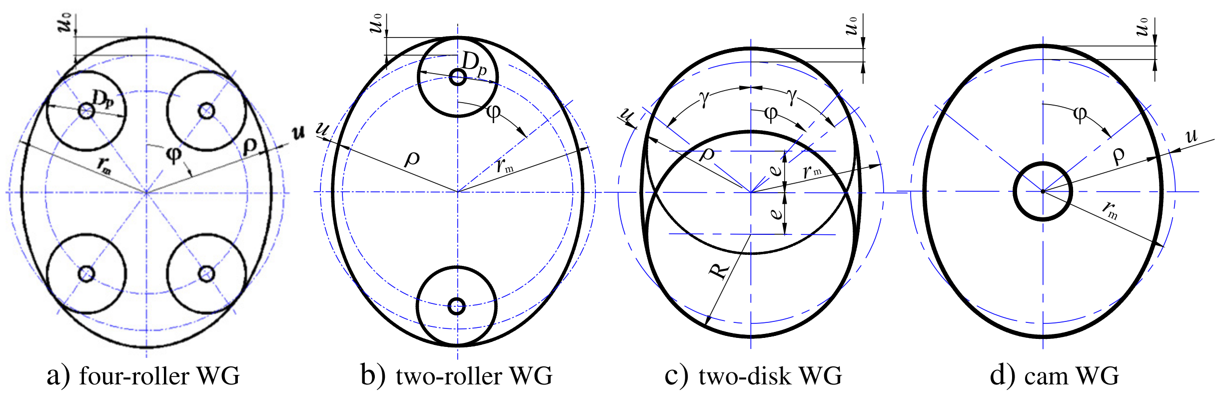

Fig. 1. Four kinds of diagram of conventional mechanical WG.