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Analytically evaluating the influence of crack on the mesh stiffness of a planetary gear set

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A B S T R A C T

Time-varying mesh stiffness, caused by the change of tooth contact number and contact position, is one of the main sources of vibration of a gear transmission system. In order to comprehensively understand the vibration properties of a planetary gear set, it is necessary to evaluate the mesh stiffness effectively. When a crack happens in one gear, the mesh stiffness will decrease and consequently the vibration properties of the gear system will change. This change of vibration can be characterized through dynamic simulation of a gearbox and processed further to detect the crack severity and location. In this paper, the potential energy method is used to analytically evaluate the mesh stiffness of a planetary gear set. A modified cantilever beam model is used to represent the external gear tooth and derive the analytical equations of the bending, shear and axial compressive stiffness. A crack propagation model is developed and the mesh stiffness reduction is quantified when a crack occurs in the sun gear or the planet gear.

1. Introduction

When a pair of spur gear meshes, the tooth contact number and the tooth mesh position change during meshing. It leads to a periodic variation in the gear mesh stiffness. The mesh stiffness variation is one of the main sources of vibration in a gear transmission system [1]. Crack may occur in gears due to excessive service load, inappropriate operating conditions or simply fatigue [2]. When a crack takes place, the gear mesh stiffness will reduce and consequently the vibration characteristics of the gear system will change. If the stiffness reduction can be quantified for different crack levels, the corresponding vibration signal can be obtained through dynamic simulation. The vibration signal can be processed further for crack detection and prognosis. Both the finite element method (FEM) and the analytical method (AM) have been used to evaluate the gear mesh stiffness [2–4]. But, FEM is complicated and time consuming. While AM can offer a simple and effective way to evaluate the time-varying mesh stiffness.

In this paper, we use external–*external* gears to denote a meshing gear pair which contains two external gears. Similarly, we define external–*internal* gears as a meshing gear pair which contains one external gear and one internal gear. Many researchers have applied the analytical method to evaluate the mesh stiffness of a pair of fixed shaft external–*external* gears. Yang and Lin [6] proposed the potential energy method to calculate the mesh stiffness of a pair of external–*external* spur gears by considering Hertzian contact stiffness, bending stiffness and axial compressive stiffness. Later, Tian et al. [7] introduced an additional term called the shear stiffness in the potential energy method. Recently, Zhou et al. [5] took the deformation of the gear body into consideration. The gear tooth was modeled as a cantilever beam which started from the base circle [5–7]. Also, Zhou et al. [5], Tian et al. [7], and Pandya and Parey [8] considered that the gear crack follows a linear path starting from the point of intersection of the base circle and the involute curve as shown in Fig. 1. Actually, the gear tooth starts from the root circle rather than the base circle as given in Fig. 1. Thus, their models ignored the gear tooth part between the root circle and the base circle. The tooth profile of this part (tooth fillet area) is not an involute curve and it is basically determined by cutting tool tip trajectory. Using a different cutting tool tip trajectory, the generated curve will be different and there is not a uniform function to depict it [10]. However, the

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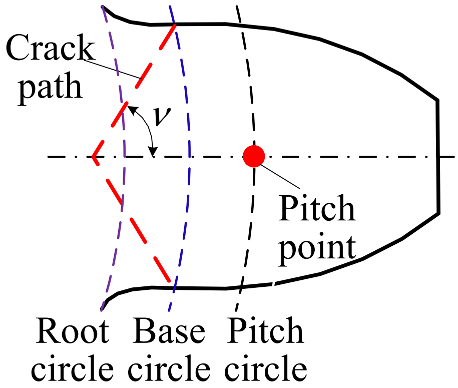

Fig. 1. Crack model from Refs. [5,7].

ignorance of this part will change the stiffness of the meshing gears, especially when the distance between the base circle and root circle is bigger. Chaari et al. [11] presented an analytical method to evaluate the mesh stiffness of a pair of external–*external* gears and modeled the crack as a straight line starting from the root circle. They mentioned that the gear mesh stiffness can be calculated by taking into account the tooth thickness reduction. However, they did not provide the analytical equations of the mesh stiffness for a crack gear. Chen and Shao [12] proposed an analytical mesh stiffness model with tooth root crack propagating along both the tooth width and the crack depth. Further, they [13] investigated the effect of the tooth profile modification on the mesh stiffness. Their model is more realistic as compared to other models. However, their analytical equations are not convenient to use. Their method was based on single point estimation of the mesh stiffness. In this case, for time-varying stiffness measurement, it is required that information at every point on the gear are known beforehand which is repeated in nature for different gears and time consuming. In this paper, a model is proposed which solves this problem and does not need variable information at every point. Rather, a model is developed which can calculate all these information by itself and use it to determine the time-varying stiffness. The analytical equations proposed in this paper are much easier to use. Given the gear geometries and the material properties, the mesh stiffness can be expressed as a function of the angular displacement of the gear.

For a planetary gear set, there are pairs of sun–planet gears (external–*external* gears) and pairs of ring–planet gears (external–*internal* gears) meshing simultaneously. Chaari et al. [14] and Walha et al. [15] used a square waveform to approximate the time-varying mesh stiffness of a planetary gear set. In their method, the amplitudes of the sun–planet mesh stiffness and the ring-planet mesh stiffness were assumed without a specific qualification method. Besides, the square waveform reflects only the effect of the change in the tooth contact number, but ignores the effect of the change in the tooth contact position. Liang et al. [16] evaluated the mesh stiffness of a perfect planetary gear set using the potential energy method. They also treated the sun gear and the planet gear as a cantilever beam starting from the base circle. It is mentioned in Ref. [14] that the amplitude modulation can be used to obtain the mesh stiffness of a planetary gear set with crack. Fig. 2 [14] illustrated the amplitude loss of 50% due to a crack in the sun gear. But, they modeled the stiffness reduction only in the double tooth contact duration while ignored the stiffness decrease in the single tooth contact duration. Also, physical meaning of this loss was not described, like how much crack propagation will lead to the amplitude loss by 50%. In this study, we propose an approach to overcome these shortcomings.

In this paper, the mesh stiffness of a planetary gear set is analytically evaluated using the potential energy method. To model the external gear, i.e. the sun gear and the planet gear, a modified beam model is proposed by considering the gear tooth starting from the root circle. Further, a crack propagation model is developed and the mesh stiffness equations are derived when a crack takes place in the sun gear or the planet gear. This crack model can be used for the detection of the crack severity and the crack

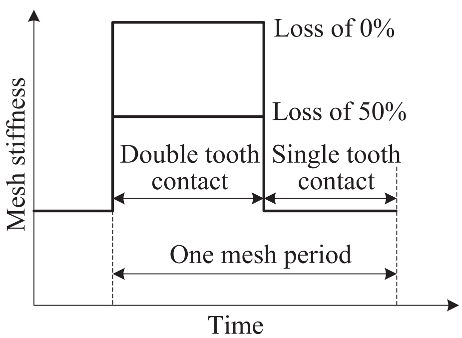

Fig. 2. Amplitude loss of 50% due to a crack in the sun gear [14].