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A semi-analytical load distribution model for side-fit involute splines

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ABSTRACT

A semi-analytical load distribution model for side-fit involute splines has been proposed by extending the formulations of elastic bodies in contact. The spline compliances considered in the model consist of tooth bending and shear, tooth base flexibility, tooth contact, and torsional deformations. This model allows input of arbitrary loading components and arbitrary initial separations. With this, load distribution characteristics of splines under various combined loading conditions can be determined. In addition, the effects of design variations and manufacturing errors can also be quantified. Aside from the versatility of the model in terms of its capabilities, a new multi-step discretization solution scheme is devised to reduce computational effort thus qualifying the semi-analytical model as a design tool. Numerical results for an example spline under various loading conditions including: (i) pure torsion, (ii) combined torsion and radial loads, and (iii) combined torsion, radial loads and tilting moments, are presented and compared to those predicted by a deformable-body model for the verification of the results.

1. Introduction

Involute splines are widely used in mechanical drive systems to transfer rotary motion and torsion from one rotation component to another, say from a shaft to a gear or vice versa. A spline joint consists of an external and an internal spline of the same number of teeth that are tightly fitted together along the same rotational axis. The spline teeth have similar profile shapes as that of involute gear teeth, while they typically have higher strength than that of gear teeth due to their larger pressure angles and shorter tooth heights. Despite this, failures of spline joints due to fretting wear, fretting corrosion fatigue, and tooth breakage occur frequently. All these failures of splines within their design life cannot be thoroughly understood unless the distributions of load along its teeth are known. In the absence of a model to conveniently predict load distribution of splines, addressing spline durability issues is often based on trial-and-error or component tests.

Review of the literature reveals only a few analytical spline load distribution models, all of which were limited to simple loading conditions. This is due to the complexity of contact at the spline interface where contacts occur over a wide area along engaged spline teeth and all spline teeth are potentially in contact simultaneously. Volfson [1] proposed a rough estimation of contact force distribution along the axial direction of splines under pure torsion or pure bending loading conditions. Tatur and Vygonnyi [2] developed an analytical model to estimate torque distribution along the face width direction of spline teeth for the case when the spline joint carries pure torsion. Barrot et al. [3–5] formulated the spline tooth torsional stiffness in the model of Ref. [2] by analyzing spline tooth deflections due to tooth bending, shear, compression, and base rotation. They calculated the load distribution along the axial direction of splines under pure torsion loading conditions. Recently, Wink and Nakandakar [6]

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proposed a model to calculate the load sharing of spline teeth under spur gear loading condition. These analytical models provide an estimate of load distribution along the face width direction of the splines under simple loading conditions, but they fail to predict the load distribution across the spline tooth profile direction. They also fall short of handling load distribution of splines under arbitrary combined loading conditions. Other complicating effects such as spline tooth surface modifications and manufacturing errors such as indexing errors are also not considered in these models.

Another group of more recent studies proposed computational models of splines using the finite element (FE) method or boundary element (BE) method. A number of studies [7–10] used commercial FE packages to predict spline load distributions under pure torsion loading, while the last two accounted for the effects of certain manufacturing errors as well. The FE models for helical spline couplings were proposed by Leen et al. [11–13] and Ding et al. [14–16] for splines under combined torsional and axial loading. Adey et al. [17] proposed a BE model for the analysis of spline joints. Their model had the capability to handle combined torsional and bending loading in the presence of certain manufacturing errors. Using Adey's model, Medina and Olver [18,19] studied load distribution of misaligned splines, and the impact of spline pitch errors and lead crown modifications. Recently, Hong et al. [20] developed a deformable-body model of gear-shaft splines using a commercial gear contact analysis software. Their model was versatile in its capability of handling load distribution of splines under combined loading conditions and with different types of surface modifications, manufacturing errors and shaft misalignment. While these computational models are superior to any analytical model in capabilities and accuracy, they require considerable computational time for each analysis. In some situations, analysis of a spline joint requires multiple contact analysis at different rotational positions, spanning an entire revolution. In such cases, the same contact analysis must be repeated at a large number of rotational positions, which makes the use of these computational models rather impractical.

This paper aims at developing a computationally efficient and accurate semi-analytical model for prediction of spline load distributions. First, the formulation of a new spline load distribution model will be presented. Next, the main components of spline compliance formulations will be described. Then, a new multi-step discretization solution scheme will be proposed. Finally, load distributions of splines under various nominal loading conditions including pure torsion loading, spur gear loading, and helical gear loading conditions will be investigated using this semi-analytical model and compared to those predicted by the deformable-body model of Ref. [20] for the verification of the model.

2. Semi-analytical spline contact model

The contact between spline joints under combined torsion, radial loads, and tilting moments can be interpreted as a general elastic contact problem with spline bodies having multi degrees of freedom. A mathematical programming method proposed by Conry and Seireg [21] has been shown to be effective and efficient in treating elastic bodies in contact. However, their model was limited to loading conditions where each elastic body has only one degree of freedom. This section aims at developing a new spline load distribution model as a general elastic contact problem that can be solved mathematically. The major assumptions of the proposed spline contact formulation are as follows:

- All deformations are elastic and the total elastic deformation is the sum of various components of elastic deformations.
- The elastic deformations are small compared to the size of splines such that the surface curvatures of splines over the contact zone can be assumed to remain unchanged.
- Contact forces on one tooth will induce no elastic deformation on adjacent spline teeth except for the torsional deflections.
- Effect of any friction between engaged spline tooth pairs is negligible.

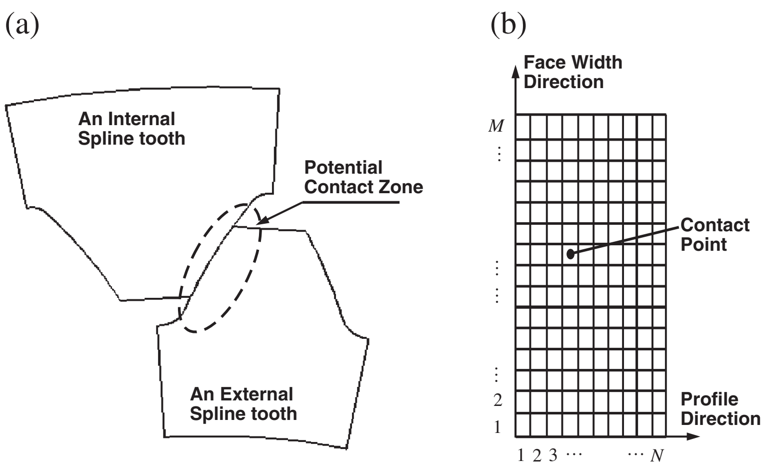

Fig. 1. (a) Schematic side view of potential contact zone of a spline tooth pair, (b) Discretization of the potential contact zone into a set of contact grids.