Probabilistic Engineering Mechanics 29 (2012) 7–15

Contents lists available at SciVerse ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

An improved approximation for the spectral representation method in the simulation of spatially varying ground motions

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ARTICLE INFO

Article history:
Received 11 August 2011
Received in revised form
30 November 2011
Accepted 21 December 2011
Available online 29 December 2011

Keywords:
Ground motions simulation
Spectral representation method
Spatially varying
Approximation

ABSTRACT

The spectral representation method (SRM), based on the Cholesky decomposition of either cross spectral density matrix or lagged coherency matrix, is widely used in the simulation of spatially varying ground motions. In this study, the SRM, based on the decomposition of lagged coherency matrix, is modified to apply to the common case which the auto spectral densities of simulation points are not the same. When using interpolation approximation approach to improve the efficiency, the SRM based on the decomposition of lagged coherency matrix exhibits much higher accuracy than the SRM based on the decomposition of cross spectral density matrix, because the elements of lower triangular matrix obtained by the Cholesky decomposition of lagged coherency matrix vary slowly with the frequency. Therefore, the SRM, based on the decomposition of lagged coherency matrix, is generally suitable for the combination with the interpolation approximation approach.

1. Introduction

Real records of strong motion array show that the ground motions vary within a small scale. When analyzing dynamic responses of structures, uniform input is acceptable for small structures, but not for large structures such as dams, bridges, tunnels and buried pipelines. In fact, large structures are very sensitive to the spatial variability of the earthquake ground motion. Therefore, it is necessary to use spatially varying ground motions as input to analyze the dynamic problems of large structures.

The effect of spatial variability of earthquake ground motions on the responses of large structures has been studied [1–12]. Among these studies, the frequency domain using stochastic method and response spectrum method are usually adopted. Others were on the basis of the time domain analysis. However, both stochastic method and response spectrum method accept the linear hypothesis. This assumption is not valid for most of the structures which are relatively flexible and behave nonlinearly because of either the geometrical or dynamic effect. In such cases, time domain analysis is essential to use. Consequently, one must require seismic ground motions simulated at locations across the structure span, which are correlated in both temporal and spatial domains [13].

Simulation of spatially correlated earthquake ground motions by the theoretical seismological approach is very complex. This approach requires detailed knowledge of fault sizes, rupture mechanisms, propagation paths (reflections, refractions), distance from the epicenter, and local geological and topographical conditions, and these data are usually not fully available [14]. Therefore, the stochastic approach proves useful for practical applications.

Many stochastic methods are proposed to simulate spatially varying seismic ground motions, which can be grouped into conditional simulation [13–16] and unconditional simulation [17–24]. Some of these stochastic methods are systemically summarized by Zerva [1]. Although many simulation methods are available, the spectral representation method (SRM) is one of the popular methods.

Rice [25] characterized a stochastic process using a spectral representation for the first time. Later, Shinozuka [26,27] applied the SRM to the simulation of Nd-1v or Nd-Nv homogeneous or non-homogeneous stochastic processes. Hao et al. [17] earlier used the SRM to simulate spatially varying ground motions. Shinozuka and Deodatis [28] summarized the characteristic of the spectral representation method in the simulation of Nd-1v stochastic processes. Actually, the SRM is nonergodic in the simulation of 1d-Nv stochastic processes, thus Deodatis [29] further extended

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doi:10.1016/j.probengmech.2011.12.001

the spectral representation method to generate stochastic ergodic sample functions by incorporating into the double-indexing frequency. Deodatis [19] used the SRM to simulate spatial variate seismic ground motions consistent with the target response spectrum. Bi and Hao [24] extended the SRM to simulate spatially varying ground motions at sites with varying conditions.

However, the main drawbacks of the SRM result in expense in memory and time, especially under conditions of the large number of simulation points or the frequency steps. Thus, it is important to increase the simulation efficiency of the algorithm. The simulation efficiency depends mainly on the decomposition of the matrix and superposition of trigonometric functions. Based on these two aspects, related studies tried to improve the simulation efficiency.

The Fast Fourier Transform (FFT) technique [30] was widely employed to improve the superposition efficiency of trigonometric functions. To improve the decomposition efficiency, Yang et al. [31, 32] gave the closed form solution of the Cholesky decomposition of cross spectral density matrix to avoid the repetitive decomposition of cross spectral density matrix in wind field simulation, when the simulation points were distributed uniformly and the coherency functions were exponential functions of distance. Cao et al. [33] extended this solution to simulate the ergodic wind field. However, this solution is usually limited in the simulation of seismic ground motions, because above two conditions may not be satisfied. Ding et al. [34] reduced the amount of cross spectral density matrix for Cholesky decomposition by using the interpolation approximation approach in wind field simulation; hence less computer memory and elapsed time are realized.

The SRM combined with interpolation approximation approach can also be applied in the simulation of spatially varying ground motions. When using the interpolation approximation approach, one question occurs: Is this scheme reliably to meet the high accuracy?

The SRM, based on the Cholesky decomposition of lagged coherency matrix, is modified to apply to the common case which the auto spectral densities of simulation points are not the same. Afterward, this paper investigates the numerical accuracy when using the interpolation approximation approach.

2. Descriptions of ground motion field

The variations in the ground motion mainly result from the following sources [35]: (i) loss of coherency of seismic waves due to scattering in the heterogeneous medium of the ground, as well as due to the differential superposition of waves arriving from an extended source, collectively called as the "incoherence effect"; (ii) difference in the arrival times of waves at separate stations, called as the "wave-passage effect"; (iii) spatially varying local soil profiles and the manner in which they influence the amplitude and frequency content of the bedrock motion underneath each station as it propagates upward, called as the "site-response effect"; and (iv) gradual decay of wave amplitudes with distance due to geometric spreading and energy dissipation in the ground medium, called as the "attenuation effect".

In practice, spatially varying ground motions can usually be considered as a 1D-NV zero mean stationary Gaussian processes $\boldsymbol{V(t)}$, which consists of N components $v_1(t), v_2(t), \dots, v_N(t)$. In the probabilistic viewpoint, the relationship among the components can be presented by the cross covariance matrix or cross spectral density matrix.

The cross covariance matrix is expressed as:

$$\mathrm{R_{VV}}(\tau) = \begin{bmatrix} R_{11}(\tau) & R_{12}(\tau) & \cdots & R_{1N}(\tau) \\ R_{21}(\tau) & R_{22}(\tau) & \cdots & R_{2N}(\tau) \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1}(\tau) & R_{N2}(\tau) & \cdots & R_{NN}(\tau) \end{bmatrix}_{N\times N}, \tag{1}$$

where τ is time lag; $R_{jj}(\tau)(j = 1, 2, \dots, N)$ are auto correlation functions of the components and $R_{jk}(\tau)(j, k = 1, 2, \dots, N, j \neq k)$ are the corresponding cross correlation functions. Based on the stationary hypothesis, the following relations are satisfied:

$$R_{jj}(\tau) = R_{jj}(-\tau), \quad j = 1, 2, \dots, N \tag{2a}$$

$$R_{jk}(\tau) = R_{kj}(-\tau), \quad j = 1, 2, \dots, N; j, k = 1, 2, \dots, N; j \neq k. \tag{2b}$$

And the cross spectral density matrix is given as:

$$\mathrm{S_{VV}}(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1N}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1}(\omega) & S_{N2}(\omega) & \cdots & S_{NN}(\omega) \end{bmatrix}_{N\times N}, \tag{3}$$

where $S_{jj}(\omega)(j = 1, 2, \dots, N)$ are real-defined auto spectral densities of the N components, and $S_{jk}(\omega)(j, k = 1, 2, \dots, N, j \neq k)$ are the corresponding cross spectral densities, which are usually defined as:

$$S_{jk}(\omega) = \gamma_{jk}(\omega)\sqrt{S_{jj}(\omega)S_{kk}(\omega)}, \tag{4}$$

where $\gamma_{jk}(\omega)$ is the coherency function, which can further be given as [35]:

$$\gamma_{jk}(\omega) = \gamma_{jk}(\omega)^{\text{incoherence}}\gamma_{jk}(\omega)^{\text{wave passage}}\gamma_{jk}(\omega)^{\text{site response}} = \left|\gamma_{jk}(\omega)\right| e^{i[\theta_{jk}^{w}(\omega)+\theta_{jk}^{s}(\omega)]}, \tag{5}$$

where $|\gamma_{jk}(\omega)|$ is the lagged coherency function, conforming the following relationship:

$$\left|\gamma_{jk}(\omega)\right| = \left|\gamma_{kj}(\omega)\right|. \tag{6}$$

$\theta_{jk}(\omega)^w$ is phase change due to "wave-passage effect", which is defined as:

$$\theta_{jk}(\omega)^w = -\omega d_{jk}^L/v_a, \tag{7}$$

where v_α is the apparent wave velocity and d_{jk}^L is the projection of d_{jk} on the ground surface in the direction of propagation of seismic waves, and we have:

$$d_{jk}^L = -d_{kj}^L \tag{8a}$$

$$d_{ij}^L - d_{ik}^L = d_{kj}^L. \tag{8b}$$

When considering the above relationship of d_{jk}^L, the following relations concerning $\theta_{jk}(\omega)^w$ hold:

$$\theta_{jk}(\omega)^w = -\theta_{kj}(\omega)^w = -\theta_{jk}(-\omega)^w = \theta_{kj}(-\omega)^w \tag{9a}$$

$$\theta_{ij}(\omega)^w - \theta_{ik}(\omega)^w = \theta_{kj}(\omega)^w. \tag{9b}$$

And $\theta_{jk}(\omega)^s$ is the phase change due to "site-response effect", which is defined as:

$$\theta_{jk}(\omega)^s = \tan^{-1}\frac{\mathrm{Im}[H_{gj}(\omega)H_{gk}(-\omega)]}{\mathrm{Re}[H_{gj}(\omega)H_{gk}(-\omega)]}, \tag{10}$$

where $H_{gj}(\omega)$ and $H_{gk}(\omega)$ are the frequency response functions of the first filter corresponding to point j and point k respectively, shown in Eq. (28). Therefore, the following relations relating to $\theta_{jk}(\omega)^s$ are:

$$\theta_{jk}(\omega)^s = -\theta_{kj}(\omega)^s = -\theta_{jk}(-\omega)^s = \theta_{kj}(-\omega)^s \tag{11a}$$

$$\theta_{ij}(\omega)^s - \theta_{ik}(\omega)^s = \theta_{kj}(\omega)^s. \tag{11b}$$

Taking into account Eqs. (6), (9) and (11), the following relations about $S_{jk}(\omega)$ can be derived:

$$S_{jk}(\omega) = S_{jk}^*(-\omega) = S_{kj}^*(\omega) = S_{kj}(-\omega), \tag{12}$$

where the subscript "*" denotes the complex conjugate.