Probabilistic Engineering Mechanics 29 (2012) 53–63

Contents lists available at SciVerse ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Probability distribution of energetic-statistical strength size effect in alpine snow

D.M. McClung [a,*], C.P. Borstad [b]

[a] Department of Geography, University of British Columbia, 1984 West Mall, Vancouver, B.C. V6T 1Z2, Canada

[b] Department of Civil Engineering, University of British Columbia, 2324 Main Mall, Vancouver, B.C. V6T 1Z1, Canada

ARTICLE INFO

Article history:
Received 24 June 2010
Received in revised form
24 July 2011
Accepted 17 August 2011
Available online 7 September 2011

Keywords:
Probability density function
Quasi-brittle fracture
Alpine snow
Avalanches
Weibull theory
Gamma distribution

ABSTRACT

Alpine snow in which avalanches form is a quasi-brittle material with an energetic (or fracture mechanical) strength size effect. However, there is also a probabilistic aspect to the size effect. In this paper, we present field data for unnotched tensile fracture of uniaxially stressed samples and 3 point bending tensile experiments from a cold laboratory. In addition, we summarize field data on shear fracture tests of avalanche weak layers. Taken together, the argument is derived from about 1300 tests, 90% of which are from field data. On the basis of the data, as well as simple theory relevant to unnotched quasi-brittle fracture, we suggest that the tensile and shear fracture data follow a gamma probability density function (pdf). For our data, the gamma pdf is essentially equivalent to a normal pdf as in Daniels' fibre bundle model for the implied distribution shape factors. However, the physical description of the failure process to derive the gamma pdf for alpine snow differs substantially from that in Daniels' classical fibre bundle model.

1. Introduction

It has been known for a long time that dry alpine snow displays a strength size effect in tension [1,2] and in shear [3,4]. In addition, there is an important density effect on strength since typical alpine snow densities of interest in avalanche formation range from about 100 to 500 kg/m^3 which implies the volume fraction filled by ice grains from 10% to 55%. In fact, the increase of fracture strength with density is much larger than changes due to the size effect for typical sample sizes [5,6] since a larger sample normally implies higher density. This is due to alpine snow densification under its own weight. Therefore, in order to study the size effect on tensile fracture of alpine snow, it is important that the samples have the same density and that they come from the same layer to avoid variations with snow structure, temperature and crystal form. To our knowledge, all the previous studies on probabilistic snow strength size effect did not follow this prescription and we suggest that the previous results [1,2,7] are inaccurate or invalid for this reason due to the lack of care in experimental technique.

There are two basic types of energetic size effects which apply to alpine snow in regard to avalanche formation. Snow slab avalanches initiate within a weak layer underneath a cohesive slab as a mode II fracture precipitated from a macroscopic imperfection [8–10]. This type of fracture (starting from an imperfection, notch or crack) follows a type 2 strength size effect as defined by Bažant [11]. Type 2 fractures are not significantly subject to material randomness [11,12]. The other type of energetic size effect is the tensile failure of the crown of a slab avalanche which fails at crack initiation and is equivalent to an unnotched sample without pre-existing cracks. This is designated as a type 1 size effect [5,11]. Both weak layer failures and slab tensile fracture are treated in this paper using unnotched test data for samples of small size. In this paper, we present tensile fracture data from 3 sets of experiments. Each set is taken from a single snow layer with nearly constant density and same size for each sample. We then analyse each data set separately with respect to the probability density function implied by the fracture strength. The data include 2 sets from uniaxial tensile failure from field data and one set from precision 3 point beam (3PB) tensile tests measured in a cold lab. The shear test data are from hundreds of shear frame experiments in different avalanche weak layers.

When the principles of quasi-brittle fracture are applied to the data and test geometry, we conclude that the data most likely follow a gamma pdf. The gamma pdfs derived are essentially equivalent to normal pdfs since the shape parameter is so high for each data set. Thus, for practical purposes either a gamma or normal pdf could be used for specifying the strength size effect for unnotched tensile fracture of alpine snow. However, since the normal pdf also appears in the classical bundle model of Daniels [13], we show that it would not be correct to specify the size effect law for alpine snow as following Daniels model.

For alpine snow, the quasi-brittle failure mode is physically different from the Daniels model and the probabilistic prediction will also differ. In type 1 quasi-brittle fracture, catastrophic fracture

* Corresponding author. Tel.: +1 604 822 9157; fax: +1 604 822 6150.
E-mail addresses: mcclung@geog.ubc.ca (D.M. McClung), cborstad@gmail.com (C.P. Borstad).

0266-8920/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.probengmech.2011.08.009

of the sample occurs when a crack is initiated from a representative volume element (RVE) which has a length scale on the order of the size of the fracture process zone (FPZ) for the material [14]. Fracture is not due to progressive failure of elements with load redistribution among the survivors across the entire sample as in the classical bundle model nor is it due to the random natural occurrence of the weakest flaw as with Weibull statistical theory. The theory employed here to derive the gamma pdf combines elements of both of these classical statistical theories but the physical process differs from both.

The other important case is the shear strength of avalanche weak layers. For that case, we summarize unnotched shear fracture tests from shear frames. We conclude that for small sizes, the pdf most likely follows a gamma distribution. Application of the theory in our paper suggests that, for very large sizes, the randomness associated with small size samples disappears, i.e. the differential entropy approaches zero.

The unnotched fracture data (more than 1300 samples in tension and shear) imply a consistent trend for either failure mode. Small samples follow a gamma or normal pdf with a failure probability decreasing with increasing sample size. For very large sizes, tensile fracture strength might follow a Weibull pdf but verification would be very difficult or impossible. Homogeneous layers (slabs) with thickness greater than about 20–30 cm rarely, if ever, exist in alpine snow. However, for thin avalanche weak layers in shear, we show it is more likely that macroscopic imperfections make the process deterministic (type 2 quasi-brittle fracture). Since the effective sample size is essentially infinite for this case, macroscopic imperfections will likely control the fracture process [8–10].

2. Unnotched tensile fracture data

The tensile fracture data used in this paper consist of three sets, all with very close control on snow type to ensure that density, temperature, crystal forms and micro-structure did not vary from sample to sample. The data consist of two sets of uniaxial data (37 and 28 samples) from in-situ tests for two separate layers in an alpine snow pack and one data set from unnotched 3 point beam (3PB) tests all from the same layer measured in a cold lab (28 samples). The latter were tested immediately after having been brought in the lab without storage. The field data (two sets) were taken with all samples fractured in succession on the same day. The experimental technique has been explained previously [15,16] for the field data and for the lab data [6]. All the basic strength data used in this study consist of nominal tensile strength. For example, with the field data, the samples were shaped so that a stress concentration factor of about 2.1 was present but not accounted for in calculating the tensile strength. The values used here were simply taken as the nominal strength without the stress concentration factor.

The strain rate for the field data is estimated at about 10^{-3}/s and it is about 10^{-1}/s for the lab data. Since alpine snow is a rate dependent material, it is to be expected that the tensile strength values will exhibit rate dependence. However, since all samples in each set were fractured in the same manner, we expect that the rate dependence will not affect the results here since we are studying the pdf for the sets of samples separately. Table 1 contains descriptive statistics for all three data sets. In Table 1, the descriptive statistics are tabulated in relation to the ratio of nominal strength (σ^*) to the mean value ($\overline{\sigma}$) for the data set. Data set 1 (uniaxial) had a mean nominal strength 2.0 kPa with snow density 216 kg/m^3, data set 2 (uniaxial) had mean nominal strength 4.7 kPa with snow density 251 kg/m^3 and data set 3 (3 point beam bending) had mean nominal strength, 21.5 kPa, and snow density 185 kg/m^3.

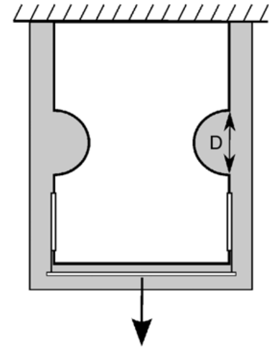

Fig. 1a. Schematic of the field test set up for uniaxial tensile strength experiments from Ref. [15]: data sets #1 and #2.

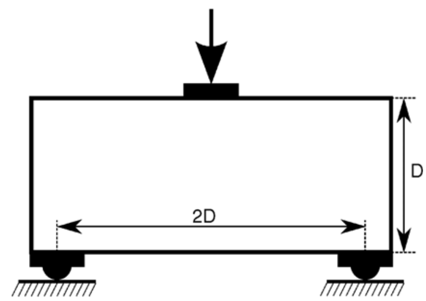

Fig. 1b. Schematic of test set up for unnotched 3 point beam tensile strength experiments in the lab: data set #3.

The mean value, $\overline{\sigma} = \overline{\sigma}(D, \rho, T, \dot{\varepsilon}, structure)$ where $\rho, T, \dot{\varepsilon}$, *structure* are snow density, temperature, strain-rate and snow structure including grain size and hardness. It is imperative to keep these variables constant in testing to isolate the sample size effect (D).

It is shown in Appendix A.2 that density increases with D in avalanche applications will account for much greater increases in strength than decreases with size D.

The field data [15,16] had sample length 0.1 m, sample width 0.2 m and sample thickness approximately 0.2 m, the latter which varied with layer thickness. The lab data had slab depth 0.1 m, span between supports 0.2 m and sample thickness 0.1 m. Thus, for the field data, the length of the sample in tension was 0.1 m and for the lab data, the approximate length scale is the span width (0.2 m) considering tensile stress in the outer fibre of the beam. In avalanche terminology, D is the crown height (or slab depth) of the tensile fracture line of a slab avalanche. Thus, for our data, the equivalent scale is: $D = 0.1$ m for both the uniaxial and the lab experiments. The sample length perpendicular to the applied tensile force is D for the uniaxial experiments and approximately $2D$ for the lab experiments. Fig. 1 contains schematics of both types of experiments.

For the 3PB lab data, the span to depth ratio is 2 which means that they are classified as deep beams. For such a span to depth ratio, the simple beam theory (pure bending) does not apply exactly since there will be different stresses than from the simple theory. If the beams were to be regarded as elastic, brittle all the peak strength values would have to be reduced by about 9% from the simple beam theory for a span to depth ratio of 2. This may easily be shown by the correction factor given on page 281 of [17]. This has not been done here for two reasons. First, alpine snow is not brittle and elastic. Second and more important is that, in this paper, the two quantities used are $\sigma^*/\overline{\sigma}$ and the COV. In both cases, such a constant multiplier on strength would not affect the ratios.