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Obtaining configuration space and singularity maps for parallel manipulators

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A B S T R A C T

The aim of this paper is to describe a general methodology to obtain the entire set of positions that a parallel manipulator can reach and the workspace regions where the robot is controllable. The workspace is computed using a hybrid analytical-discrete procedure. Next the singularity maps are traced by carrying out a kinematic analysis of the positions obtained. To perform the latter a systematic method has been introduced to obtain the corresponding Jacobian matrices. The result of the whole process is the computation of singularity-free workspace regions, associated with certain working and assembly modes. After that, strategies to enlarge the accessible space are easier to plan and implement. This methodology is based on disassembling the manipulator into a mobile platform and a set of kinematic chains.

1. Introduction

The workspace is considered one of the major design factors of parallel robots. Therefore, it is very important to have an efficient tool capable of determining the workspace, which may contain internal voids, in addition to its external boundary. Also, kinematic singularity loci, that may divide the space, have to be assessed. A detailed analysis of the workspace is necessary to carry out efficient path planning.

The representation of workspaces is normally given by the set of positions reached by a point of interest on the MP, i.e., the TCP. When the pose of the MP must be considered, orientation angles have to be added to that representation and depicting it becomes more complex. In parallel manipulators with more than three DOF only partial representations are possible, these are obtained by constraining as many position variables as exceed this number [1–3]. According to the choice of parameters to represent, different kinds of subspaces can be considered [4], namely:

- Constant orientation workspace: set of points accessible to the TCP while maintaining a specific orientation of the MP [5].
- Reachable (or Maximal) workspace: points located by the TCP whatever orientation of the MP.
- Dexterous workspace: points accessed by the TCP where all orientations of the MP can be reached.
- Total orientation workspace: positions where the MP can reach all the orientations within given ranges.
- Inclusive orientation workspace: positions where the MP can reach at least one orientation within given ranges.
- Orientation workspace: set of MP's orientations for a given position of the TCP [6,7].
- Operational (or useful) workspace: set of postures the manipulator can reach without blockade of actuators [8].
- Joint space: workspace projected onto the input variables domain.

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Nomenclature

MP	mobile platform
TCP	tool center point
KC	kinematic chain, in the sense of limb or *leg* of the parallel manipulator
IKP	inverse kinematic problem
DKP	direct kinematic problem
$\lvert \mathbf{J}_{IKP} \rvert$	inverse Jacobian
$\lvert \mathbf{J}_{DKP} \rvert$	direct Jacobian
DOF	degrees of freedom

To obtain workspaces, different methods may be considered; essentially discretization methods, geometric methods or analytical methods. Discretization methods [9–12] consist in establishing a mesh of nodes where TCP positions and/or MP orientations have to be checked, normally by means of solving the IKP. Their main advantages are their easy computational implementation and the possibility of modeling all kinds of constraints. The most noteworthy disadvantages are their computational cost and a precision dependent on the mesh step size. Some other variants are based on using interval analysis [13], where discrete postures are substituted by variable intervals.

Geometrical methods [14–16] are widely used to determine constant orientation workspace boundaries. They are based on first obtaining the surfaces defining the reach limits of the TCP considering the MP attached only to one KC and, subsequently, identifying the intersection for all KCs. Their main disadvantage is that the computation is usually limited to the constant orientation workspace, and also the use of additional geometric tools (e.g., CAD) is usually required, to perform boolean operations on geometric entities.

Analytical methods [17–19] consist in posing an optimization problem with penalties on workspace boundaries. Most of the analytical approaches are strongly dependent on the robot architecture, thus these methods lack a capacity for generalization and they may be considered most useful for application to specific manipulators.

Likewise, it is essential to perform a complete analysis of singularities in the foregoing workspace before a robot path planning. On the one hand, IKP singularities occur at workspace boundaries. In these postures a dependence among the output parameters happens restricting output capability, which is equivalent to an instantaneous reduction in the number of DOF of the MP. However, reaching an IKP singularity does not imply losing control on the manipulator. On the other hand, DKP singularities occur inside the workspace [20,21]. At these singular postures a dependence relation among the input joint-rates occurs, which implies the robot becoming uncontrollable. Therefore, it is essential to find the singularity locus and some proximity indicators to define operational workspaces.

In addition, there are some effective approaches to compute the singularity-free workspace for 2-DOF parallel manipulators, such as the enveloping curve based approach [22]. The method presented here can be also applied to singularity-free n-DOF fully-parallel manipulators. For solving the workspace and determining the singularity maps CAD variation approaches are also effective [23,24]. For some manipulators singularity loci can be obtained in closed form, but as these are usually analytical approaches, they are heavily dependent on the robot architecture. The methodology described in this paper will be able to solve singularities in a general way.

2. Scope of the procedure

The procedure introduced in this paper covers the finding of workspaces in parallel manipulators with uncoupled freedoms, i.e., resorting to the theory of group of displacements, all manipulators whose MP has a motion represented by a displacement subgroup. Hence, the aforementioned method will not be able to analyze parallel manipulators with lower mobility and mixed freedoms. These are characterized by types of motion where there is a dependency relation among output variables that will require constraint analysis. However, this latter can be implemented if those constraints are found in advance; it being difficult to solve algorithmically.

Regarding the type of method developed according to the classification mentioned above, this procedure can be placed between discretization and analytical methods. The configuration space is a $2n$-dimensional continuum (or a set of them) in the variables considered (n being the number of DOF of the robot), and the workspace is a subspace, or a set of disjoint-subspaces. In general-purpose applications, it is often convenient to approximate it by a set of discrete positions. Thus the procedure makes an approximation of the actual continuum by virtue of a discretization of this domain. On each point, however, all calculations will be analytical. Thus, the method can be described as a hybrid analytical-discrete procedure.

In essence, the method implemented has been divided into two fundamental steps. First, every position in the workspace is found within a given discrete resolution. Second, all these postures of the manipulator are subjected to a kinematic analysis, where Jacobian matrices are computed, norms obtained and the singularity loci determined by virtue of a further refinement of the mesh used.