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# Workspace density and inverse kinematics for planar serial revolute manipulators

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ABSTRACT

The topic of reachable workspaces of robotic manipulators has received considerable attention over the past half century. One approach to generating workspaces is by sampling joint angles and evaluating the boundary of the resulting set in the space of rigid-body motions. In the case when the manipulator has discrete actuation, such as stepper motors or pneumatic cylinders, not only the boundary of the workspace, but also the density of reachable poses within the workspace is important. Following previous efforts that focused on characterizing this workspace density, we show that this density is particularly efficient to evaluate in the special case of planar serial arms with revolute joints. We then show how the resulting density can be used in inverse kinematics algorithms that are equally applicable for discrete-state and continuous-motion robot arms.



## 1. Introduction

Planar robots are currently used in a number of industrial and medical applications [1]. Moreover, commonly used industrial architectures such as SCARA manipulators like the one shown in Fig. 1 contain planar manipulators as critical components.

The development of algorithms for highly accurate and stable control of planar robotic arms is therefore an important topic. The solution of the inverse kinematics problem is a fundamental part of robot control. Traditionally, three models have been used to solve the inverse kinematics problem. The first is the geometric model, which is well-suited to compute the inverse kinematics of relatively simple manipulators with a small number of links. For example, A. Yu et al. [2] published a geometric approach to the accuracy analysis of class 3-DOF planar parallel robots. The second is the algebraic model, which does not guarantee a "closed-form" solution, but can be efficiently solved by polynomial root finding. D. Manocha and J.F. Canny [3] presented an algorithm and implemented it for efficient inverse kinematics for a general 6R manipulator by extending the polynomial elimination methods of M. Raghavan and B. Roth [4]. The third is the iterative model, the result of which depends on the starting point used. This approach to finding the inverse kinematics solution of robotic manipulators was proposed by J.U. Korein and N.I. Badler [5].

In parallel with intelligent control developments, there are additional novel approaches for solving the inverse kinematics problem. For example, neural-network-based inverse kinematics solution methods for robotic manipulators have been explored recently in [6–12]. For example, B. Karlik and S. Aydin [6] presented a structured artificial neural-network (ANN) to the solution of inverse kinematics problems for a six-degree-of-freedom robot manipulator. Work has been undertaken to find the best ANN configurations for this problem. J.A. Martin et al. [7] proposed a method to learn the inverse kinematics of multi-link robots by evolving neuro-controllers. The method is based on the evolutionary computation paradigm and obtains incrementally better neuro-controllers. R.V. Mayorga and P. Sanongboon [8] presented an ANN approach for fast inverse kinematics computation and effective geometrically bounded singularity prevention of redundant manipulators. E. Oyama et al. [9] proposed a novel expert

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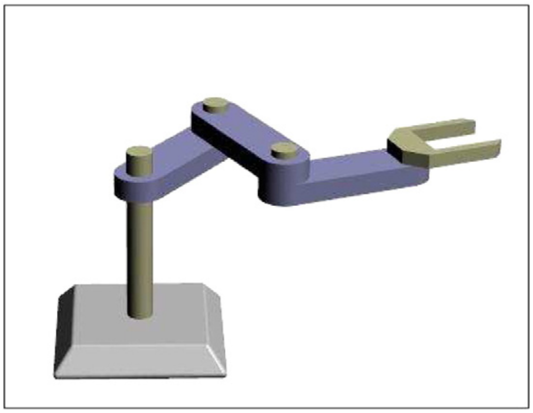

**Fig. 1.** A SCARA manipulator.

selection by using performance prediction networks which directly calculate the performances of the experts which could reduce the computation time. S.S. Chiddarwar and N.R. Babu [10] presented a fusion approach to determine inverse kinematics solutions of a six degree of freedom serial robot. The effectiveness of the fusion process was shown by comparing the inverse kinematics solutions obtained for an end-effector of an industrial robot moving along a specified path with the solutions obtained from conventional neural network approaches as well as an iterative technique. S. Tejomurtula and S. Kak [11] presented an ANN approach for solving the inverse kinematics problem. The method yielded multiple and precise solutions. It was suitable for real-time applications. S.K. Nanda et al. [12] proposed a novel application of ANNs for the solution of inverse kinematics of robotic manipulators. This method represents the non-linear mapping between Cartesian and joint coordinates using multi layer perceptrons and a functional link artificial neural network.

Genetic algorithm approaches, such as in Refs. [1,13], and [14], have been widely investigated. P. Kalra et al. [1] presented an approach based on an evolutionary genetic algorithm that was used to obtain the solution of the multimodal inverse kinematics problem of industrial robots. A.C. Nearchou [13] used a modified genetic algorithm to search successive robot configurations in the entire free space to specify how the robot should move its end-effector. R. Köker [14] presented a genetic algorithm approach to a neural-network-based inverse kinematics solution for robotic manipulators based on error minimization. In that work, ideas from neural network algorithms and genetic algorithms were fused.

The Jacobian pseudo-inverse approach is a widely used method for solving the inverse kinematics problem. A.A. Maciejewski and C.A. Klein [15] presented the Singular Value Decomposition (SVD) of the Jacobian to compute pseudo-inverse for robotic manipulators. R.G. Roberts and A.A. Maciejewski [16] presented repeatable control strategies that obtain near optimal solutions in the selected workspace.

Researchers have also focused on some other approaches to obtain inverse kinematics solution for robot manipulators. B. Siciliano [17] addressed the inverse kinematics, manipulability analysis, and closed-loop direct kinematics algorithm for the Tricept robot. H. Zhang [18] presented a method to compute inverse kinematics in parallel for robots with a closed form solution. The computational task of inverse kinematics was partitioned with one subtask per joint and all subtasks were computed in parallel. This results in effectiveness and the efficiency of the algorithm for a multiprocessor system. S.R. Lucas et al. [19] compared the merits of many of the methods already presented and described a new approach that led to a fast and numerically well-conditioned algorithm. P. Chiacchio et al. [20] presented new closed-loop schemes for solving the inverse kinematics of constrained redundant manipulators. G.S. Chirikjian and J.W. Burdick [21] presented efficient kinematic modeling techniques for “hyper-redundant” robots. Their methods were based on a backbone curve that captures the hyper-redundant robot's important macroscopic features. I. Ebert-Uphoff and G.S. Chirikjian [22] introduced algorithms for inverse kinematics of discretely actuated hyper-redundant manipulators using workspace densities. They proposed a framework for the discussion of the discretely actuated case and presented an inverse kinematics algorithm. This builds on prior sampling-based approaches to manipulator workspace analysis such as in Refs. [23,24] by observing that sampled workspaces of subchains can be smoothed to result in densities, and these densities can be “added” by convolution to result in the density for the whole manipulator. Y. Wang and G.S. Chirikjian [25] presented workspace generation of hyper-redundant manipulators as a diffusion process. In that work, the evolution of the workspace density function is defined by a diffusion equation, which depends on manipulator length and kinematic properties. A multi-objective optimum design of general 3R manipulators for prescribed workspace limits was proposed by M. Ceccarelli and C. Lanni [26]. In that paper a suitable formulation for the workspace was used for the manipulator design, which was formulated as a multi-objective optimization problem by using the workspace volume and robot dimensions as objective functions, and given workspace limits as constraints.

Each methodology contains certain advantages and disadvantages for solving the inverse kinematics problem of robot manipulators. Our paper derives the “closed-form” workspace density and inverse kinematics for planar serial revolute robot arms.