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Kinematic and power-flow analysis of bevel gears planetary gear trains with gyroscopic complexity

Germano Del Pio, Ettore Pennestrì *, Pier Paolo Valentini

Dipartimento di Ingegneria dell'Impresa, University of Rome Tor Vergata, via del Politecnico, 1 00133 Roma, Italy

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ABSTRACT

In this paper the authors propose a method for the kinematic and power-flow analysis of bevel epicyclic gear trains with gyroscopic complexity. By gyroscopic complexity, we mean the possibility of the gear carrier to be a floating link as, for instance, in robotic gear wrists.

Thanks to the new formulas herein deduced, the methods based on the graph representation of planetary spur gear trains have been extended to bevel gear trains. In particular, the well known Willis equation has been modified to maintain its validity for bevel gears. The modified Willis equation was then embodied in new power ratio expressions. Under our simplifying hypotheses of absence of friction and constant angular speeds, it is shown that gyroscopic torques do not enter into power flow analysis. Two numerical examples are discussed.

1. Introduction

In recent times different papers on the mechanics of multi degrees-of-freedom planetary gear trains have been published. The focus of this effort was the development of systematic methods for mechanical efficiency analysis. This renewed interest is also due to the application of epicyclic gear trains as one of the main powertrain component of hybrid vehicles. The capability to handle torques from different power sources is an almost unique feature of this type of transmissions.

A fundamental step in mechanical efficiency analysis is the ascertainment of the amount of power flow through the meshing gears. Although not self evident, due to power circulation, some meshing gears may sustain a power higher than the input one. Power circulation, that usually occurs with very low transmission ratios, must be detected at the early design stages in order to dimension properly meshing gears and lubricating methods.

Most of the contributions are related to spur gear trains. In this case the kinematics can be studied with the classic scalar Willis equation.[1] The relationship between the absolute angular speeds of bevel gear trains is not scalar and this complicates the analysis.

This paper focuses on kinematic and power flow analysis of planetary trains with bevel gears. It can be considered as an attempt to extend the *modus operandi* of the analysis methods devised for spur gear trains to bevel gear trains.

A complete review of all the scientific contributions on the topic is outside the purpose of this paper. Thus the following review cannot be considered exhaustive.

Belfiore, Pennestrì and Sinatra [1] presented a Maple procedure for kinematic and power-flow analysis of spur planetary gear trains based on the graph based method of Pennestrì and Freudenstein [2,3].

* Corresponding author. Tel.: +39 0672597138.
E-mail address: pennestri@mec.uniroma2.it (E. Pennestrì).

[1] Following the tradition of many textbooks, the authors called “Willis equation” the expression which relates the absolute angular velocities of a differential gear train. However, it should be acknowledged that this equation was well known before Rev. Robert Willis (1800–1875), Jacksonian Professor of Applied Mechanics in the University of Cambridge.

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Kaharaman et al. [4] proposed a modular general formulation valid for 1-dof automatic transmission composed of complex–compound planetary gear sets.

Chen and Teh [5] applied the concept of virtual-power [6] to find ready-to-use formulas for the mechanical efficiency analysis of 2-dof gear trains. For a Simpson differential gear train, Chen [7] analyzed the sensitivity of the mechanical efficiency.

Mathis and Remond [8] proposed a unified model for the kinematic, torque and efficiency analysis of epicyclic gear trains. An application of the method to a Ravigneaux type gear train is discussed.

Galvagno [9] discussed the influence of dynamics on the mechanical efficiency analysis of a 2-dof differential spur gear train.

Pennestrì et al. [10] presented a systematic approach for the modeling and analysis of power split transmissions which include an epicyclic gear train. The method is a refinement of the one proposed by Pennestrì and Freudenstein [2,3,11].

However, less common are the power flow and efficiency analyses of gear trains with bevel gears.

Day et al. [12] proposed a matrix method for the kinematic analysis of planetary bevel gear trains using the concept of the fundamental circuit. However, the method is limited to gear carriers rotation about a fixed axis.

Freudenstein et al. [13] extended the concept of fundamental circuit to the analysis of geared robotic wrists. This category of wrists, enumerated by Belfiore [14], has gyroscopic complexity when the gear carrier is not adjacent to the frame link.

Further contributions on planetary geared robotic wrists are due to Tsai [15] who hinted that the motion of a bevel-gear-type end-effector can be described by an equivalent open-loop chain. The analysis equations follow by considering relative rotation between every two adjacent links in the equivalent open-loop chain and coaxial conditions.

Litvin and Zheng proposed a matrix method for the kinematic analysis of differential trains with bevel gears [16].

Gupta and Ma [17], extended the tabular superposition method to derive the relations among the three wrist joint variables and three coaxial actuation variables.

Nelson and Cipra [18] proposed a systematic graph-based matrix method for kinematic, power-flow and mechanical efficiency suited to the complete solution of bevel gear sets as well as planar epicyclic sets.

Uyguroglu and Demirel [19,20] applied oriented linear graph techniques toward the kinematic and static moment analysis of robotic bevel-gear trains.

Staicu proposed recursive matrix relations for kinematics and dynamics analysis of different orienting bevel gear train [21–23]. The relations are particularly useful for inverse dynamics.

Laus et al. [24] combined graph and screw theory for the analysis of mechanical efficiency of bevel gear trains with complex architecture. In particular Davies' equations [25] have been applied to gear trains. The analysis equations are deduced in a way similar to the one followed for electrical networks. The mechanical analog of the electrical resistance is introduced to take into account power losses.

Chen [26] introduced the constraint equations for kinematics and power flow analysis. The method is very systematic and can be potentially implemented in a general purpose code. The use of constraint equations for kinematic and dynamic analysis of planetary gear trains was also explored by Mantriota and Pennestrì [27] by means of multibody dynamics approach.

The classical tabular method, based on Willis equation, cannot be applied to complex planetary bevel gear trains.

In this paper, we propose a scalar equation between the absolute angular velocities of the simplest bevel gear train. By means of this equation, the method of fundamental circuits [28,29] can be extended to the kinematic analysis of planetary bevel gear trains.

Moreover, the power ratios through the bevel gears and gear carrier forming a fundamental circuit are herein deduced. The result is also novel. In fact, such ratios include, as a particular case, those deduced by Pennestrì and Freudenstein [2,3] for planetary spur gear trains.

By means of these ratios, graph based methods of power flow analysis of planetary gear trains can include the presence of bevel gears.

The paper is divided into three parts. The first two parts are dedicated to methods of kinematic and power flow analysis, respectively, and the third one to applications.

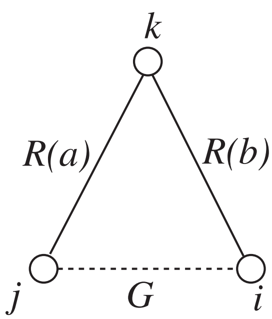

Fig. 1. Labeled graph representation of a fundamental circuit. G: Gear pair; $R(a)$, $R(b)$: revolute pairs with different axes; i and j, gear wheels; k gear carrier (transfer vertex).