Mechanism and Machine Theory 69 (2013) 350–372

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Inverse kinetostatic analysis of compliant four-bar linkages

Nicola Pio Belfiore *, Pasquale Simeone

Sapienza University of Rome, Dept. Mechanics and Aerospace Engineering, via Eudossiana, 18, 00149 Rome, Italy

ARTICLE INFO

Article history:
Received 31 July 2012
Received in revised form 15 April 2013
Accepted 21 June 2013
Available online 18 July 2013

Keywords:
Compliant mechanisms
Curved beam
Displacement matrix
Four-bar linkage
MEMS

ABSTRACT

A new method is proposed for the inverse kinetostatic analysis of compliant four-bar linkages with flexible circular joints and *pseudo-rigid* bodies. The theory of curved beams has been applied to the flexible parts and a novel closed form symbolic expression is presented for the compliance matrix *C*, which maps the generalized forces and relative displacements for the free section of the curved beam with respect to the framed one. This matrix has been checked by using commonly use Finite Element Analysis package, with good agreement. Then, the theory of planar displacement matrices has been applied in order to solve the static balance of the whole system. The method could be applied also to different mechanisms with different topologies and different flexural pivots, providing that the compliance matrix *C* and the whole system balance equations are properly modified. Results presented herein are believed to be useful for two reasons: firstly, because it is possible to determine quickly the values of the whole sets of externally applied forces for a given pose, under quasi-static condition; secondly, because such a fast method allows analyzing several series of adjacent poses. In the paper, how this opportunity has been sized in order to achieve new knowledge about the modeling of a compliant four-bar linkages via pseudo-rigid-body model approximation has been described.

1. Introduction

Compliant mechanisms have been successfully adopted in several applications [1] because their design and fabrication are made easier by some valuable features: a) they can be built as a unique block of material; b) there is a neutral configuration where the system holds a stationary pose; c) backlash is absent; and d) lubrication can be eliminated.

Apart from the experimental methods there are three major means to analyze deformations in compliant mechanisms, namely, Finite Element Analysis (FEA), continuum mechanics or lumped-parameter model simulation. Each one of these methodologies has its characteristic advantages and drawbacks so they are adopted depending on the case study. For example, FEA offers the possibility of a detailed analysis of strain and deformation in the whole structure, but requires proper packages or environments, together with a certain demand for computational time and space. Continuum mechanics provides elegant closed form or numerical formulations for many classes of problems, but these solutions will be basically dedicated to specific geometry or a restricted class of basic structures. On the other hand, lumped-parameter models are very easy to analyze but they are rather approximated and may be inaccurate, especially for large deformations or for complex mechanisms.

This investigation is dedicated to compliant four-bar linkages composed of 4 pseudo-rigid links, one of which is the frame link, and 4 flexural joints, which are supposed to be made of a constant curvature constant rectangular cross-section curved beam. The flexural characteristics of the curved beams will be analyzed by means of continuum mechanics and will be validated by FEA. The inverse kinetostatic problem will then be solved by imposing the balance of both links and joints and by using the displacement matrices theory. Finally, this new and fast procedure will be used to evaluate the forces and moments required to accomplish

* Corresponding author. Tel.: +39 0644585227; fax: +39 06484854.
E-mail address: belfiore@dima.uniroma1.it (N.P. Belfiore).

0094-114X/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechmachtheory.2013.06.008

three different prescribed motions around the neutral configuration. As described later, this calculation will provide some knowledge about lumped-parameter modeling.

The method based on concentrated parameters herein deserves a brief consideration, for the sake of a better description of the present investigation. In the realm of *compliant mechanisms*, the lumped-parameters are all, or, almost all, based on the so called *pseudo-rigid-body* model which can be built, starting from the compliant system, in two steps. Firstly, the *compliant mechanisms* is partitioned in sub-parts, each one belonging either to the set of *pseudo-rigid* links or to the set of flexible joints. Then, with reference to the plane [2] mechanisms, the *pseudo-rigid* parts are definitively regarded as rigid links, while the flexible parts are replaced by traditional kinematic pairs together with linear or torsional springs; this step is quite acceptable when the flexible parts are both thinner and shorter than the rigid ones [1]. Such a drastic subdivision into *rigid* or *flexible* sets can be justified herein by the circumstance that the flexural stiffness of rectangular constant cross-section beam, in the plane, is proportional to the third power of the beam's height. Hence, if a beam has double or triple height with respect to an adjacent one, then the former will be 8 or 27 times more *rigid* than the latter. Many issues concerning *pseudo-rigid-body* model classification and conceptual definitions of *segments* and *flexural pivots* have been disclosed in a fundamental paper [3] published in 1994.

Flexible joint replacement is a crucial aspect in order to build a pseudo-rigid-body model. Assuming that any flexible part of the compliant mechanism could be modeled as a flexible beam, a certain relative motion between the beam ends is possible and the relative displacements will depend on the load condition. Unfortunately, it is not generally true that such relative motion is coincident with a revolution around a fixed rotation center. Hence, if any flexible sub-part is replaced by one single flexural pivot (R substitution) some inaccuracy may occur, while the spring stiffness value must be optimized. The evaluation of the equivalent spring stiffness for use in a pseudo-rigid-body model has been described in 1996 [4], while pseudo-rigid-body model for initially curved segments has been studied in 2001 [5].

Any flexible sub-part of the compliant four-bar is therefore supposed to be substituted. However, each substitution can be done not only by using one single flexural pin R, but also by using a series of 2 or 3 close flexural pivots, RR or RRR. Since such strategy offers more degrees of freedom (DOF) it is possible that RR or RRR substitution imitates the real beam ends' relative motion better than single R substitution. However, RR or RRR substitution will be much more complicated to use for analysis and synthesis purposes. In any case, the use of single flexural pivot transforms the compliant mechanisms into a lumped-parameter model which is very easy to study. In fact, dealing with a discrete system is an approach that is much more advantageous than studying a continuous domain. For example, thanks to the pseudo-rigid-body modeling, the Theory of Kinematic Synthesis of mechanisms can be conveniently adopted in order to synthesize a mechanism designed for a given function, specially path and function generation for infinitesimal displacements [6,7]. A dedicated approach to the synthesis of planar, compliant four-bar linkage has been presented in 2001 [8]. However, it is not so easy to find a proper method to identify the best fitting pseudo-rigid-body equivalent mechanism, because R, and even RR, substitutions are structurally unable to offer the 3 degrees of freedom (DOF) that are required, in the plane, in order to reproduce any possible relative motion between the beam ends, while RRR substitution is very complicated and needs three spring stiffness constants and, at least, two geometric ratios to be defined.

Among the large amount of contributions dedicated to the compliant mechanisms, some have been dedicated to the accurate modeling of the flexible sub-parts. For example, in [9] a four-link pendulum with flexible straight line joints has been modeled and optimized by means of distributed-parameter method. The pendulum has been assumed to be composed of rigid bodies jointed together by means of straight flexible beams and the elastic elements deformations have been evaluated via Euler–Bernoulli model. In [10] a fully compliant bistable micromechanism has been analyzed by using straight flexible joints and some experimental results have been provided for comparison. A fair complete account about the behavior of several kinds of flexible joints is available in many other valuable contributions [11–16], where the accuracy of stress and deformation evaluation methods is also investigated. The accuracy of some equations that can be adopted for the study of flexible beams has been also compared with respect to FEA in 2008 [17].

Different kinds of flexible joints have been adopted in order to design linkage mechanisms [18,19], magnification devices for precision mechanisms [20], spherical parallel robots [21], parallel micromanipulators [22–24], and microgrippers [25].

In [26] the problem of synthesis has been considered in relation to the presence of compliant joints and therefore the synthesis equations have been coupled with the equations of the joint elasticity by assuming the latter being regulated as if the joints were torsional springs connecting two pseudo-rigid members.

Distributed compliance mechanisms have also been investigated, as, for instance, in [27,28], where new design methods have been suggested.

Finally, lumped and curved beams have been evaluated as possible elements to be used as flexible joints between pseudo-rigid bodies. Both kinds have been compared with the straight line beam in [29].

In the present paper the flexible sub-parts will be assumed to have the shape of a constant cross-section homogeneous and isotropic curved beam, having initially a circular axis. In fact, the original scope of this work consisted in the development of a tool for supporting the analysis of a new concept flexural hinge [30,31] which takes some advantages from the curved shape.

Curved beams have been studied in literature. For example, in [32] stress and deformations have been evaluated with the Timoshenko and Euler–Bernoulli models, while in [33] the approach has been based on a two node Finite Element model which relies on the use of the potential energy principle and of the Hellinger–Reissner functional.

In [34] an analytical treatment has been proposed for the evaluation of inextensible curved beam large deformations together with a closed form expression of the beam stiffness. In [35] a similar analysis has been performed by means of polar coordinates. A more complex method, which is based on the solution of systems of differential equations, in global Cartesian coordinates, has been presented in [36]. Finally, in [37] an exact formulation has been obtained by means of the Hamiltonian State Space.