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Feedback minimization of the first-passage failure of a hysteretic system under random excitations

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ABSTRACT

The stochastic optimal bounded control of a hysteretic system for minimizing its first-passage failure is presented. The hysteretic system subjected to random excitation is firstly replaced by an equivalent nonlinear non-hysteretic system. The controlled non-hysteretic system is reduced to a one-dimensional controlled diffusion process by using the stochastic averaging of the energy envelope method. The dynamical programming equations and their associated boundary and final-time conditions for the problems of maximization of reliability and mean first-passage time are formulated. The optimal control law is derived from the dynamical programming equations and the control constraints. The dynamical programming equations for the maximum reliability problem and the mean first-passage time problem are finalized and solved numerically. Finally, numerical results are worked out to illustrate the application and effectiveness of the proposed method.

1. Introduction

In a control process, many structural systems incorporated with passive base isolation devices or electro/magneto-rheological dampers often exhibit hysteretic behavior when subjected to severe dynamic loading such as wind and earthquake. Many civil engineering structures also exhibit hysteresis when subject to severe dynamic loading. Therefore, the random vibration of hysteretic systems has been a significant subject in structural dynamics and it has been extensively studied [1–3]. Among methods used in the study of random vibration of hysteretic systems, the stochastic averaging of the energy envelope method, or quasi-conservative averaging, has some remarkable advantages. By using this method, the dimension of the equation of motion is reduced and more accurate results can be obtained [4–7].

In the past 20 years, the active control of structural systems has been extensively investigated. The active control of hysteretic structures under random loading is a challenging task due to the complicated behavior of hysteretic systems. The sliding mode controller and optimal polynomial controller to control hysteretic structures under earthquake excitation have been proposed by Yang et al. [8,9]. However, these controllers are deterministic and independent of dynamic loading. In the last couple of years, a nonlinear stochastic optimal control strategy has been proposed by Zhu and his co-workers [10,11] based on the stochastic averaging method for quasi-Hamiltonian systems [12,13] and the stochastic dynamical programming principle [14]. The strategy has been applied to randomly hysteretic systems [15] and extended to nonlinear stochastic optimal control of Preisach hysteretic systems recently [16]. It is more effective and efficient than linear stochastic optimal control strategy for linear stochastic systems. Furthermore, using the stochastic averaging method in the control strategy simplifies the dynamical programming equation, reduces the dimension of the dynamical programming equation and renders analytical prediction of the responses of uncontrolled and controlled systems. Thus, the proposed control strategy is very promising and deserves further development.

Feedback control is mostly used to alleviate the response of mechanical and structural systems and is sometimes used to stabilize the systems. In this paper, an optimal feedback control is designed to minimize the first-passage failure of hysteretic systems. A controlled hysteretic column subjected to both horizontal and vertical random ground motion excitations is taken as a paradigm to illustrate the optimal bounded control. The maximum reliability and mean first-passage time of the hysteretic system are obtained at the same time.

2. Hysteretic system

Consider a controlled hysteretic column subject to both horizontal and vertical random ground excitations. The equation of

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motion of this system is of the following form [17]:

$$\ddot{X} + 2\zeta\dot{X} + [\alpha - k_1 - k_2\eta(t)]X + (1-\alpha)Z = \xi(t) + u \quad u \in \bar{U} \tag{1}$$

where X is the non-dimensional displacement, ζ is the viscous damping ratio, α is the ratio of stiffness after yield to stiffness before yield, and k_1 and k_2 are constants. $\zeta(t)$ and $\eta(t)$ are horizontal and vertical ground motion excitations that act as external and parametric excitations, and they are idealized as independent Gaussian white noise with intensities $2D_1$ and $2D_2$, respectively. u is a control force, and $\bar{U}$ is the admissible set of the control force. The objective of the present study is to find the optimal control force u^* to maximize the reliability and the mean first-passage time of the hysteretic system. Z is the hysteretic component of the restoring force, which is dependent on the time history of the system and is modeled by a nonlinear differential equation [2]:

$$\dot{Z} = A\dot{X} - \beta\dot{X}\,|Z|^n - \gamma\left|\dot{X}\right| Z\,|Z|^{n-1} \tag{2}$$

where A, β, γ and n are hysteresis parameters. The smoothness of the force–displacement curve is controlled by n, the general slope of the curve is controlled by $\gamma+\beta$, and the slimness of the hysteresis is controlled by γ. By adjusting the values of these hysteresis parameters, a variety of hysteretic restoring forces such as softening or hardening and low or high dissipation can be constructed to model the actual hysteresis behavior. To apply the stochastic averaging of the energy envelope method, the hysteretic system (1) can be replaced by the following equivalent nonlinear non-hysteretic stochastic system:

$$\ddot{X} + [2\zeta + 2\zeta_1(H)]\dot{X} + \partial U(X)/\partial X = \xi(t) + k_2 X\eta(t) + u \tag{3}$$

where

$$H = \dot{X}^2/2 + U(X). \tag{4}$$

H is the total energy of the system and U is the equivalent potential energy. The nonlinear damping coefficient is obtained by the energy dissipation equivalence in one cycle as follows:

$$2\zeta_1(H) = \frac{A_r}{2\int_{-a}^{a}\sqrt{2H-2U(X)}\,\mathrm{d}X} \tag{5}$$

where A_r is the area of the hysteresis loop; a is the amplitude of the displacement and is related to H by $H = U(\pm a)$. The expressions of the potential energy $U(X)$ and hysteresis area A_r depend on the values of the hysteresis parameters. In the case $n = A = 1$ and $\gamma = \beta$,

$$U(X) = \frac{1}{2}(\alpha - k_1)X^2 + \frac{1}{2}(1-\alpha)(X+X_0)^2 \quad -a \le X \le -X_0 \tag{6a}$$

$$U(X) = \frac{1}{2}(\alpha - k_1)X^2 + \frac{1}{8\gamma^2}(1-\alpha)(1-\mathrm{e}^{-2\gamma(X+X_0)})^2 \quad -X_0 \le X \le a \tag{6b}$$

$$A_r = (1-\alpha)[2X_0/\gamma - (\alpha - X_0)^2] \tag{7}$$

in which X_0 is the residual hysteretic displacement. The quantities a and X_0 can be obtained for given H by solving the following equations:

$$2\gamma(a - X_0) = 1 - \mathrm{e}^{-2\gamma(a+X_0)} \tag{8a}$$

$$2H - (\alpha - k_1)a^2 = (1-\alpha)(a-X_0)^2. \tag{8b}$$

3. Stochastic averaging

Suppose that the difference between the energy input by excitation and the energy dissipated by damping and the control force is small compared with H in Eq. (3). In this case, H is a slowly varying process and the stochastic averaging of the energy envelope method [5] can be applied to (3); the following partially averaged Itô stochastic differential equation for the total energy H is obtained:

$$\mathrm{d}H = \left[m(H) + \left\langle \frac{\partial H}{\partial \dot{X}} u \right\rangle\right]\mathrm{d}t + \sigma(H)\mathrm{d}B(t) \tag{9}$$

where $B(t)$ is the unit Wiener process, and $\langle\cdot\rangle$ represents the averaging operator. The drift coefficient $m(H)$ and the diffusion coefficient $\sigma(H)$ are as follows:

$$m(H) = \frac{1}{T(H)}\left[-A_r - \int_{-a}^{a}\left[4\zeta\sqrt{2H-2U(X)} - \frac{2K_2^2D_2X^2}{\sqrt{2H-2U(X)}}\right]\mathrm{d}X + D_1\right] \tag{10a}$$

$$\sigma^2(H) = \int_{-a}^{a}(2D_1 + 2k_2^2D_2X^2)\sqrt{2H-2U(X)}\,\mathrm{d}X \tag{10b}$$

$$T(H) = 2\int_{-a}^{a} 1/\sqrt{2H-2U(X)}\,\mathrm{d}X. \tag{10c}$$

4. Formulation of dynamical programming equations

For most mechanical and structural dynamical systems, $H(t)$ represents the total energy of the system. Usually $H(t)$ varies randomly in the semi-infinite interval $(0, \infty)$. Suppose that the system starts with initial energy $H(0) = H_0$, which is in the normal or safe region $(0, H_c)$. The system will fail when its total energy $H(t)$ reaches the boundary H_c for the first time and it is a kind of first-passage failure. We are interested in the reliability, i.e., the probability of the system operating without failure, or the mean first-passage time. In this paper, we want to design an optimal bound feedback control to maximize the reliability and the mean first-passage time of the hysteretic system.

Suppose that the control forces are subjected to the control constraint

$$u \in \bar{U}. \tag{11}$$

The reliability function of the controlled system (1) is defined by

$$P\{H(t,u) < H_c, 0 \le t \le T\}. \tag{12}$$

Define the value function as

$$V(t,H) = \sup_{u\in\bar{U}} P\{H(t,u) < H_c,\ 0 \le t \le T\} \tag{13}$$

where sup is the abbreviation of the word "supremum". It is seen from Eqs. (12) and (13) that the value function is actually the reliability function of the optimally controlled system (9). Based on the stochastic dynamical programming principle [18], the following dynamical programming equation for the value function can be derived from system Eq. (9):

$$\sup_{u\in\bar{U}}\left\{\frac{\partial}{\partial t} + \left[m(H) + u\frac{\partial H}{\partial \dot{X}}\right]\frac{\partial}{\partial H} + \frac{1}{2}\sigma^2(H)\frac{\partial^2}{\partial H^2}\right\} \times V(t,H) = 0 \quad 0 \le t \le T,\ H \in (0, H_c). \tag{14}$$

The boundary conditions associated with Eq. (14) are

$$V(t, H_c) = 0 \tag{15}$$

$$V(t, 0) = \text{finite} \tag{16}$$