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Stochastic finite element analysis of a cable-stayed bridge system with varying material properties

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ABSTRACT

Stochastic seismic finite element analysis of a cable-stayed bridge whose material properties are described by random fields is presented in this paper. The stochastic perturbation technique and Monte Carlo simulation (MCS) method are used in the analyses. A summary of MCS and perturbation based stochastic finite element dynamic analysis formulation of structural system is given. The Jindo Bridge, constructed in South Korea, is chosen as a numerical example. The Kocaeli earthquake in 1999 is considered as a ground motion. During the stochastic analysis, displacements and internal forces of the considered bridge are obtained from perturbation based stochastic finite element method (SFEM) and MCS method by changing elastic modulus and mass density as random variable. The efficiency and accuracy of the proposed SFEM algorithm are evaluated by comparison with results of MCS method. The results imply that perturbation based SFEM method gives close results to MCS method and it can be used instead of MCS method, especially, if computational cost is taken into consideration.

1. Introduction

Bridges with very long spans have always been a great challenge for engineers throughout history. Cable-stayed types of bridges are becoming more and more popular in the construction of long span bridges due to their many advantages, i.e. light in weight, efficient in load resistance, and of smaller cross sections. Cable-stayed bridges which consist of main girders, towers and cables are complicated structures. From these towers, cables stretch down diagonally and support the girder. Cable-stayed bridge can be distinguished by the number of spans, number of towers, girder type, number of cables and types of cables. The cable-stayed bridge can be constructed for even longer spans, if the deck and cable stiffness and strength to weight ratios can be improved. This could significantly diminish the critical compressive stresses of the deck in the tower zones, and increase the apparent stiffness of the stay-cables, as their sag action is reduced due to a huge drop of the weight per unit length.

The traditional structural analyses are realized according to the assumption that geometrical and material characteristics of structures are deterministic. However, there are some uncertainties about design values. These uncertainties can be defined as geometrical characteristics (cross-internal area, flexural inertia, length etc.), material characteristics (elastic modulus, Poisson' ratio, mass density etc.), and magnitudes and distributions of the loads. The deterministic method could be disqualified for many structural system analyses because of these uncertainties. MCS is the most employed method among the stochastic analysis methods for structural problems. It lies on the generation of a defined number of samples of the uncertain parameters and on the solution of the corresponding deterministic problems. However, as the number of degrees of freedom of the structure and the number of uncertain parameters increase, structural analyses with the Monte Carlo become very heavy from a computational point of view, and, in some cases, the computational effort makes them inapplicable. Accordingly, some non-statistical alternative procedures have been proposed in the literature [1–5]. On the other hand, stochastic finite element method (SFEM), which is one of the probabilistic analysis methods, increases its reliability day by day. Most of them are based on perturbation techniques, so that the SFEM is often identified as the classical finite element method (FEM) coupled with a perturbation approach. This method is applied several field in civil engineering, especially, simple or semi-complex structure systems.

Although there is an extensive literature on deterministic analysis of bridges [6–9], technical literature is not adequate on the stochastic dynamic analysis of cable-stayed bridge. The dynamic behaviors of cable-stayed bridges have been studied by several researchers [10,11]. Linear and nonlinear static and earthquake-response analyses of cable-stayed bridges were carried out by many researchers [12–15] only in the past two decades.

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The SFEM algorithm for structures has been developed by several researchers [16–20]. However, most of their work is limited to simple structures. More complex structures such as cable-stayed bridges are not considered. Very few researchers [21–23] studied the stochastic finite element method with random variable material and geometrical properties of cable-stayed bridges. Cheng and Xiao [22] proposed a stochastic finite-element-based algorithm for the probabilistic free vibration and flutter analysis of suspension bridges through combination of the advantages of the response surface method, FEM and MCS. Liu et al. [23] investigated large flexible structures, such as suspension bridges, actually possess stochastic material properties and these random properties unavoidably affect the dynamic system parameters. It is concluded from numerical analysis of a modern suspension bridge that although the second-order statistics of frequencies are small relatively to the change of basic design variables, such as density of mass and modulus of elasticity, the sensitivities of modal parameters to these variables at different locations change in magnitude.

The focus of the present paper is to perform the stochastic dynamic analysis of a cable-stayed bridge by using the perturbation based SFEM and MCS methods. During stochastic analysis, displacements and internal forces of the systems are obtained from perturbation based SFEM and MCS methods by using different uncertainties of material characteristics. Elastic modulus and mass density are chosen as random variable material properties. The analysis results obtained from these two methods are compared with each other.

2. Stochastic finite element method (SFEM)

In the stochastic finite element method (SFEM), the deterministic finite element formulation is modified using the perturbation technique or the partial derivative method to incorporate uncertainties in the structure systems. Since the basic variables are stochastic, every quantity computed during the deterministic analysis, being a function of the basic variables, is also stochastic. Therefore, the efficient way to arrive at the stochastic response may be to keep account of the stochastic variation of the quantities at every step of the deterministic analysis in terms of the stochastic variation of the basic variables.

A SFEM, which is based on perturbation technique, is developed. The method developed here uses an alternate approach for obtaining improved computational efficiency. The derivatives of the concentration with respect to random parameters are obtained by using the derivatives of local matrices instead of global matrices. This approach increases the computational efficiency of the present method by several orders with respect to standard SFEM. There are two fundamental ways to solve the stochastic problem (i) analytical approach and (ii) numerical approach. Among analytical approaches, the perturbation method is widely used because of its simplicity. Numerical method such as Monte Carlo Simulation is generally applicable to all types' stochastic problems and is often used to verify the results obtained from analytical methods. A detailed discussion of these methods is presented below.

2.1. Perturbation based SFEM formulation

The perturbation method is the most widely used technique for analyzing uncertain system. This method consists of expanding all the random variables of an uncertain system around their respective mean values via Taylor series and deriving analytical expression for the variation of desired response quantities such as natural frequencies and mode shapes of a structure due to small variation of those random variables. The basic idea behind the perturbation method is to express the stiffness and mass matrices and the responses in terms of Taylor series expansion with respect to the parameters centered at the mean values.

Since the deterministic equations are valid for the MCS analysis as well, then the essential differences are observed in case of perturbation based stochastic finite element analysis. Let us consider a deterministic equation of motion in the form of

$$M\ddot{q} + C\dot{q} + Kq = Q_\alpha \tag{1}$$

where K, M, C denote the stiffness matrix, mass matrix and damping matrix, $\ddot{q}$, $\dot{q}$, q denote the acceleration, velocity, displacement, respectively. The stochastic perturbation based approach consists usually of up to the second-order equations obtained starting from the deterministic ones.

The basic idea of the mean based, second-order, second-moment analysis in stochastic finite element moment is to expand, via Taylor series, all the vector and matrix stochastic field variables typical of deterministic finite element method about the mean values of random variables (b), to retain only up to second-order terms and to use in the analyses only the first two statistical moments. In this way equations for the expectations and covariances of the nodal displacements can be obtained in terms of the nodal displacement derivatives with respect to the random variables.

The perturbation stochastic finite element equations describing dynamic response of random variable system for zeroth, first and second order:

Zeroth-order equation (ϵ^0 terms, one system of N linear simultaneous ordinary differential equations for $q_\alpha(b;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$M(b)\ddot{q}(b;\tau) + C(b)\dot{q}(b;\tau) + K(b)q(b;\tau) = Q_\alpha(b;\tau). \tag{2}$$

First-order equations, rewritten separately for all random variables of the problem (ϵ^1 terms, $\bar{N}$ systems of N linear simultaneous ordinary differential equations for $q_\alpha^{,\rho}(b;\tau)$, $\rho = 1, 2, \ldots, \bar{N}$, $\alpha = 1, 2, \ldots, N$)

$$M(b)\ddot{q}^{,\rho}(b;\tau) + C(b)\dot{q}^{,\rho}(b;\tau) + K(b)q^{,\rho}(b;\tau) = Q_\alpha^{,\rho}(b;\tau) - \left[M^{,\rho}(b)\ddot{q}^0(b;\tau) + C^{,\rho}(b)\dot{q}^0(b;\tau) + K^{,\rho}(b)q^0(b;\tau)\right]. \tag{3}$$

Second-order (ϵ^2 terms, one system of N linear simultaneous ordinary differential equations for $q_\alpha^2(b;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$\begin{aligned} &M(b)\ddot{q}^{(2)}(b;\tau) + C(b)\dot{q}^{(2)}(b;\tau) + K(b)q^{(2)}(b;\tau) \\ &\quad = \Big\{Q_\alpha^{,\rho\sigma}(b;\tau) - 2[M^{,\rho}(b)\ddot{q}^{,\sigma}(b;\tau) + C^{,\rho}(b)\dot{q}^{,\sigma}(b;\tau) \\ &\qquad + K^{,\rho}(b)q^{,\sigma}(b;\tau)] - \big[M^{,\rho\sigma}(b)\ddot{q}^0(b;\tau) \\ &\qquad + C^{,\rho\sigma}(b)\dot{q}^0(b;\tau) + K^{,\rho\sigma}(b)q^0(b;\tau)\big]\Big\} S_b^{\rho\sigma} \end{aligned} \tag{4}$$

where

$$q_\alpha^{(2)}(b;\tau) = q_\alpha^{,\rho\sigma}(b;\tau) S_b^{\rho\sigma} \tag{5}$$

where b is the vector of nodal random variables, q_α is the vector of nodal displacement-type variables, τ is forward time variable, $\bar{N}$ is the number of nodal random variables. M, C and K are system mass matrix, damping matrix and system stiffness matrix, respectively. Q_α, q and $S_b^{\rho\sigma}$ are load vector, displacement and the covariance matrix of the nodal random variable, respectively. N is the number of degrees of freedom in the system. $(.)^0$ is zeroth-order quantities, taken at means of random variables, $(.)^{,\rho}$ is first partial derivatives with respect to nodal random variables, $(.)^{,\rho\sigma}$ is second partial derivatives with respect to nodal random variables.

In Eqs. (2)–(4) the zeroth-order mass, damping and stiffness matrices and local vector and their first and second mixed derivatives with respect to nodal random variables b_ℓ are defined as follows;

Zeroth-order functions

$$M(b) = \int_\Omega \varphi_{\bar{\alpha}} \ell_{\bar{\alpha}}^0 \varphi_{i\alpha}\varphi_{i\beta}\,\mathrm{d}\Omega \tag{6}$$

$$C(b) = \int_\Omega \varphi_{\bar{\alpha}}\varphi_{\bar{\beta}}(\varphi_{\bar{\alpha}}^0 \ell_{\bar{\beta}}^0 \varphi_{i\alpha}\varphi_{i\beta} + \beta_{\bar{\alpha}}^0 C_{ijkl\bar{\beta}}^0 B_{ij\alpha}B_{kl\beta})\,\mathrm{d}\Omega \tag{7}$$