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Probabilistic response of linear structures equipped with nonlinear damper devices (PIS method)

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Abstract

Passive control introducing energy absorbing devices into the structure has received considerable attention in recent years. Unfortunately the constitutive law of viscous fluid dampers is highly nonlinear, and even supposing that the structure behaves linearly, the whole system has inherent nonlinear properties. Usually the analysis is performed by a stochastic linearization technique (SLT) determining a linear system equivalent to the nonlinear one, in a statistical sense. In this paper the effect of the non-Gaussianity of the response due to the inherent nonlinearity of the damper device will be studied in detail via the Path Integral Solution (PIS) method. A systematic study is conducted showing that for a very wide range of parameters the SLT gives satisfactory results in terms of variance of displacement and velocity but not in terms of joint Probability Density Function (PDF). It has also been shown that at steady state the two processes, displacement and velocity, may be considered as independent ones.

Keywords: Path integral solution; Short-time Gaussian approximation; Viscous dampers; Nonlinear systems; Stochastic linearization technique

1. Introduction

Passive control using energy absorbing devices has received considerable attention in recent years. Among them viscoelastic dampers, fluid dampers and yielding devices have been widely studied and installed as energy sinks to reduce the structural response and consequently to reduce or eliminate yielding associated to structural elements during earthquake ground motions.

In this paper, the attention is focused on the analysis of dynamic systems, equipped with nonlinear viscous damper devices. The appeal of fluid dampers is due to some interesting features: (i) they require low maintenance; (ii) they may be used for several severe earthquakes without damage; (iii) forces exerted by the damper devices are not in phase with elastic forces, and thus the former do not introduce any additional stress in the structural systems. Because of these appealing characteristics, viscous dampers have found wide application, both in the civil and military field, in shock and vibration isolation of equipment, pipe-work systems, bridges and buildings. Attempts of modeling the constitutive laws of viscous fluid dampers by using the classical theory of viscoelasticity [1], or by using the fractional derivative equations [2] may be found in the literature. Experimental and analytical investigations, on seismic response of structures filled with viscous dampers, may be found in the literature [3–5]. Since the relationship between force and velocity of the damper is highly nonlinear, even supposing that the structure behaves linearly, the whole damper–structure system has inherent nonlinear properties, due to the constitutive law of viscous damper devices, and the complete probabilistic characterization of the response process may be performed by Monte Carlo Simulations (MCS), with a great computational effort involved. An alternative technique, to overcome the difficulties arising from the nonlinearity, is the SLT, which is the most used approximate method for the analysis of nonlinear structural systems under random excitations [6–9].

The motion produced by earthquake accelerations is random in nature and so the estimation of the response statistics of linear or nonlinear systems under stochastic agencies becomes compulsory. Recently in [10] a stochastic linearization technique for the case of seismic ground motion has been proposed. For the case of normal white noise (additive or

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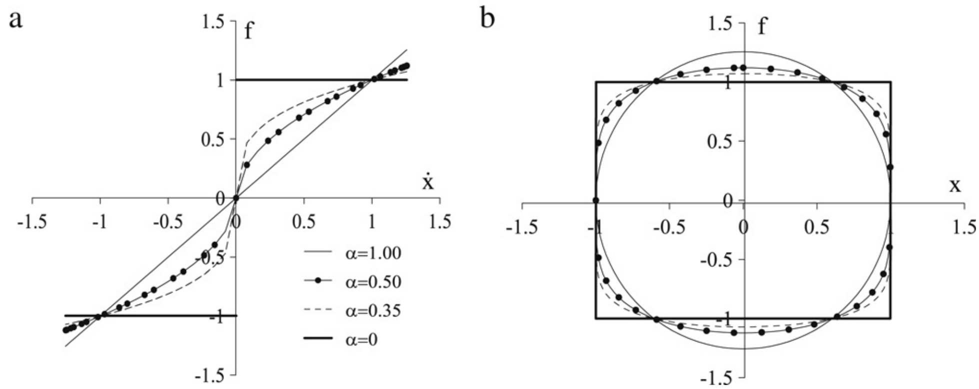

Fig. 1. Damper device constitutive law for $x(t) = \sin(\Omega t)$, for different values of α and $C_D = 1$; (a) force–velocity relationship; (b) force–displacement relationship.

multiplicative input) the response statistics may be obtained by solving the Fokker–Planck–Kolmogorov equation. However exact solutions of the partial differential equations ruling the evolution of the PDF are known only for very restricted classes [11–13]. For this reason, several approximate solution techniques have been developed, including variational methods based on eigenfunction expansion of the transition PDF [14,15], Finite Element Method [16], and so on.

Among the numerical methods for evaluating the PDF, the so-called Path Integral Solution (PIS) is an effective tool for evaluating the response in terms of PDF at each time instant, for evaluating moments of various order, energy response PDF, first passage time of strong nonlinear systems. For some strong nonlinear systems of the type $|X|$ in [17] the evaluation has been performed attesting that the PIS is reliable.

The PIS method is based on writing the Chapman–Kolmogorov equation involving the transition probability density function, where the PDF of the response at the time $(t+\tau)$ may be evaluated when the PDF at an early close time instant (t) is already known. If τ is small, even if the system is nonlinear, the transition PDF is almost Gaussian (short-time Gaussian approximation). It follows that the kernel of the integral form is Gaussian and this simplifies the analysis, as can be elicited from the application reported here (for both linear and nonlinear cases).

The aim of this paper is twofold: firstly the validity of the PIS method for nonlinearities of the type $(|\dot{X}|^{\alpha}\,\mathrm{sgn}(\dot{X}))$, that arise in structural systems equipped with viscous dampers, is confirmed by comparing results with those obtained by MCS; secondly an investigation of the joint displacement–velocity PDF is performed in order to assess the independence of displacement and velocity at steady state.

2. SLT on a single degree of freedom system equipped with a viscous damper

In this section the analysis of a single degree of freedom system equipped with a viscous damper is reported and the results obtained by SLT technique will be briefly described.

The damper effect is an output resisting force, therefore it acts in the opposite direction with respect to the relative velocity between the ends of the damper device itself. The typical force–velocity relationship is

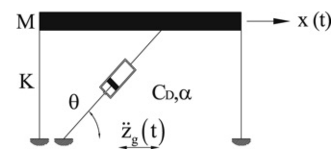

Fig. 2. One-story shear-type building model excited by ground acceleration.

$$f(\dot{x}) = C_D|\dot{x}|^{\alpha}\,\mathrm{sgn}(\dot{x}) \tag{1}$$

where $\mathrm{sgn}(\cdot)$ is the signum function, C_D and α characterizes the damper device. The value of α for seismic applications varies between 0.10 and 0.50, in this way the force rises very fast for small velocity values and becomes almost constant for large velocity values. Eq. (1) has been validated by laboratory tests performed by manufacturers and research groups. In such laboratory tests the input is a piece-wise constant velocity ranging between fixed values (with opposite signs). The force–velocity diagrams for $(x(t) = \sin(\Omega t))$ are reported in Fig. 1(a) for four different values of α. In Fig. 1(b) the corresponding force–displacement diagrams are plotted, showing the typical hysteretic behavior of the damper device.

It is worth noting that for $\alpha = 1$ the damper is linear and the force–displacement relationship gives an ellipse, while for $\alpha = 0$ the constitutive law of pure friction device is restored and the force–displacement relationship gives a rectangle.

Let us now consider the one-story shear-type building shown in Fig. 2, subjected to ground acceleration $\ddot{z}_g(t)$, modeled as a zero-mean Gaussian white noise process $W(t)$ characterized by the correlation function:

$$E[W(t_1)W(t_2)] = q(t_1)\delta(t_2 - t_1) = q(t_2)\delta(t_2 - t_1) \tag{2}$$

$E[\cdot]$ being the ensemble average, $\delta(\cdot)$ the Dirac's delta function and $q(t)$ the strength of the white noise (if $W(t)$ is stationary then $q(t) = q$).

It follows that the displacement and its derivative are stochastic processes too, and, as customary, we will denote them with capital letters, that is $x(t)$ will be replaced by $X(t)$, and so on.

Setting the global stiffness of the columns as K, the mass as M, and taking θ as the angle of the brace equipped with