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Station-keeping of moored vessels by reliability-based optimization

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Abstract

Floating marine structures are frequently kept in position by means of mooring lines in addition to a thruster system. Various types of control schemes for the thruster system are first investigated based on a simplified response model. In particular, the role of structural reliability criteria applied to the mooring system is investigated. Subsequently, a more refined control algorithm based on such reliability criteria is introduced. The performance of this control system is demonstrated by numerical simulations.

Keywords: Dynamic positioning; Thrusters; Mooring lines; Optimization; Reliability

1. Introduction

Floating marine structures are frequently kept in position by mooring lines, often also assisted by a thruster system. The operability and feasibility can be further increased by introducing an automatic control system both for dynamic positioning and reduction of dynamic response levels. The challenges related to the formulation of control schemes for such purposes are in some ways similar to those encountered in positioning and tracking of ships, see for instance Ref. [2].

It is presently focused upon configuration control with the following main purposes: (i) restricting the offset from a given reference configuration and (ii) limiting the loading on the mooring system in order to avoid failure of the mooring lines.

The control actuation is performed by means of thrusters. In the following, different types of functions are considered which reflect the cost associated with the operation of these thrusters as well as the cost associated with the vessel offset being different from zero. These explicit cost functions are compared by application to a simple quasistatic one-degree-of-freedom moored structure. Subsequently, implementation of a reliability-based control scheme for a dynamic system is addressed. An example of application is given.

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2. Simplified considerations

2.1. General

For a given type of operation (e.g. production of oil and gas), there will generally both be an associated benefit (i.e. an income, I) and loss (i.e. cost, C). The net income, N, is hence expressed as the difference between these two quantities (discounted to the same point in time): $N = I - C$.

In the present analysis, it is assumed that the first term is given. Hence, the focus is on the cost term and how this can be minimized. The cost is assumed to be composed of two different contributions: The first represents the cost associated with energy consumption by the thruster system. The second corresponds to the costs caused by the vessel offset, which implies increased probability of failure due to fatigue or extreme mooring line tension. In turn, this second cost is represented here by two mainly different types of loss functions: One of them is a quadratic function of the offset while the other function is proportional to the mooring line failure probability which in turn depends on the offset value.

Optimal control laws are subsequently derived by minimizing these loss functions. In order to achieve this in a transparent way, quasistatic response of a one-degree-of-freedom system is first considered. This implies that dynamic effects (i.e. inertia and damping forces) are neglected. Having derived these optimal control laws, dynamic systems are subsequently addressed. A control algorithm based on maintaining a target reliability level for the mooring lines is derived. In order to achieve a smoothly varying controller, this algorithm is quite different from the quasistatic case. Application of the algorithm to a moored dynamic turret system is also illustrated.

In the following, the two different types of loss functions are first introduced. As a next step, these mainly different loss functions are both implemented within a position-based (PID) control scheme and an LQG control scheme.

The first type of loss function considered is quadratic both with respect to thruster's force and (static) response:

$$L(r) = K_T F_T^2 + K_F r^2 \tag{1}$$

where K_T and K_F are (positive) proportionality factors. This type of loss function is typically applied for the derivation of LQG control schemes (on integral form), but can also be applied within a more general context.

This type of loss function is relevant for fatigue strength criteria where the damage is typically proportional to the standard deviation of the response exponentiated to a power m (which generally has values ranging from 2 to 6). The present loss function would hence be particularly relevant for cases where the value of m is 2. More general types of loss functions where the second term in (1) is replaced by $K_F r^m$ can clearly also be candidates of interest.

As an alternative type of loss function with more focus on extreme response levels, the expected cost associated with the failure of the system can be introduced. This cost hence replaces the second term of the previous function. This cost is proportional to the failure probability of the mooring line. This failure probability (corresponding to a given reference duration) for the critical mooring line is furthermore expressed in terms of the so-called delta-index as $p_f = \Phi(-\delta)$.

This index is expressed in terms of the tension for mooring line number k as

$$\delta_k(t) = \frac{T_{b,k} - T_k(r_k(t)) - g\sigma_k}{\sigma_{b,k}} \quad \text{for } k = 1 \dots q \tag{2}$$

where q is the number of mooring lines; $T_{b,k}$ is the mean breaking strength of mooring line k; $T_k(r_k(t))$ is the slowly-varying tension (i.e. static tension plus tension induced by wind and slow-drift forces); σ_k is the standard deviation of the wave-induced dynamic tension, g is a "gust-effect" scaling factor, and $\sigma_{b,k}$ is the standard deviation of the mean breaking strength.

A lower bound for δ_k is selected, denoted δ_s (which is equal for all mooring lines), that defines the critical value of the reliability index. The condition $\delta_k < \delta_s$ represents a situation where the probability of line failure is higher than a specified tolerance limit.

The loss function based on the failure probability is then expressed as

$$L(r) = K_T F_T^2 + K_P \Phi(-\delta) \tag{3}$$

where clearly both the thruster's force and the delta-index depends on the vessel offset position r.

The present expression for the failure probability represents a significant simplification. A more correct expression for the failure probability is given by

$$P(\mathsf{T}_{T\,\max} > T_{b,k})$$

where

$$T_{\max} = \max(T_k(r_k(t)) + T_{\text{Dyn}}(t)) \tag{4}$$

where $T_{\text{Dyn}}(t)$ is the dynamic wave-induced tension. The maximum value of this expression is to be taken with respect to time for a given reference period (e.g. for an extreme sea-state, for a duration of one year or for the total lifetime of the system). Both of the quantities given in the first expression are random quantities. Assessment of the failure probability can best be achieved by structural reliability methods, see e.g. Ref. [6].

However, the present purpose is to capture the effect of strength criteria on the control algorithm, and the scheme based on the application of the simplified delta-index is accordingly sufficient. (A further issue which can also be taken into account by this index, is the statistical model uncertainties related to the estimation of the dynamic tension in the mooring line. This can be achieved by modifying the denominator in Eq. (2), see e.g. Ref. [1].)

3. Position-based (PID) control in the case of quasistatic response

The quasistatic version of the dynamic equilibrium equation for a one-degree-of-freedom system (the degree-of-freedom is here taken to correspond to the surge motion of the moored vessel) is expressed as:

$$k_{\text{Tot}} r = F_E - F_T \tag{5}$$

where k_{Tot} is the linearized total stiffness of the system. A linear approximation of the mooring system will, for relatively limited deviations from the operating point (which typically is the case for dynamically positioned vessels), represent the forces from the mooring system well. F_E contains all environmental forces due to current, wind, and slow-drift forces. The control action is represented by the thruster's force F_T. The wave-induced forces and the corresponding dynamic response are both neglected in this approximation. The left-hand side of this equation is based on linearization of the force–displacement characteristics of the mooring lines at the equilibrium position.

The first-order wave-induced forces is neglected due to the present control strategy, which does not attempt to compensate for the wave-induced motion. The resulting displacement can then be expressed explicitly in terms of the forces as:

$$r = (F_E - F_T)/k_{\text{Tot}}. \tag{6}$$

As the basis for the PID control scheme, a reference position is required at each time step. In the following, the instantaneous position of the vessel for the case that no thruster forces are acting is applied as the reference position, i.e. $r_S = F_E/k_{\text{Tot}}$. In order to compute this reference position, the external force, F_E, clearly needs to be known or estimated.

Based on the vessel displacement r, the corresponding (linearized) force in the most stressed mooring line is