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Statistical tolerance synthesis with correlated variables

Isabel González [a,*,1], Ismael Sánchez [b]

[a] Department of Mechanical Engineering, Universidad Carlos III de Madrid. Avd. de la Universidad 30, 28911, Leganés, Madrid, Spain
[b] Department of Statistics, Universidad Carlos III de Madrid. Avd. de la Universidad 30, 28911, Leganés, Madrid Spain

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ABSTRACT

Optimal tolerance design aims at assigning tolerances such that the functionality requirements are achieved with minimum cost. Classical tolerance design procedures are based on the assumption of independence of variables. This assumption might not be realistic, leading to the assign of non-optimal tolerances. This paper proposes an innovative methodology to allocate optimal statistical tolerances to dependent variables, where the dependence structure is estimated from the manufacturing process. The methodology is based on the assumption that the multivariate process variability is consequence of a set of independent factors. Hence, the tolerance assignment should be based on the statistical properties of these factors. The tolerance design takes the independence of the factors as a restriction, and tolerances are optimally assigned according to the variability of them.

1. Introduction

The quality and functionality of parts are usually related with one or more variables $\mathbb{Y} = (Y_1, Y_2, \ldots, Y_J)'$, such as length, width, weight, voltage, and so on. The functionality of the part imposes some specification limits on $\mathbb{Y}$. These variables can, in turn, be a function of other variables $\mathbb{X} = (X_1, X_2, \ldots, X_K)'$. For example, the quality of a part could be defined by the variable Y_1 = maximum allowable load. This variable, in turn, depends on the length X_1, width X_2, and thickness X_3, of the part. Similarly, in a mechanical assembly, there are often certain characteristics of quality $\mathbb{Y}$, that are a function of the dimensions of the individual components that compose the mechanical assembly. In cases like these, the specification limits of $\mathbb{Y}$ imposes restrictions on the values of $\mathbb{X}$, denoted as tolerance limits. The tolerances of $\mathbb{X}$ should be assigned so that the specification limits of $\mathbb{Y}$ are met with minimum cost. In this sense, whereas specifications of $\mathbb{Y}$ are imposed by the functionality requirements, tolerances of $\mathbb{X}$ should be optimally designed. In the literature, this tolerance design of $\mathbb{X}$ is denoted as tolerance synthesis or tolerance allocation.

Tolerances of manufactured components have a significant impact on quality and manufacturing costs. Tight tolerances assures the functionality requirements of $\mathbb{Y}$ with very high probability; but it might lead to an increase in manufacturing costs. That is, too tight tolerances impose an expensive production process with very low variability. In contrast, loose tolerances may lead to increased waste and assembly problems, which lead to high costs due to lack of quality (quality costs). For this reason, tolerance synthesis is frequently formulated as an optimization problem for the optimal design of a product in terms of functionality and cost.

It is important to note that optimal tolerance synthesis is linked to the variability of the process, both with the variances and the covariances. Hence, an estimation of the covariance matrix of $\mathbb{X}$ is a critical input to reach a feasible tolerance design.

* Corresponding author.
E-mail addresses: ismael@est-econ.uc3m.es (I. González), ismael@est-econ.uc3m.es (I. Sánchez).
URL: http://turan.uc3m.es/uc3m/dpto/IN/dpin11/fabdis/imgfaria.html (I. González).
[1] Tel.: +34 916245860; fax: +34 916249430.

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This estimation of the covariance matrix of $\mathbb{X}$ has to be done with an initial sample of data of $\mathbb{X}$, in a similar fashion as in the design of control charts.

The most used method for tolerance synthesis is based on a statistical approach (statistical tolerance synthesis). Under this approach, a statistical distribution of the variables $\mathbb{X} = (X_1, X_2, \ldots, X_K)'$ is assumed as a result of the variability of the production process. An important aspect of this statistical approach is the dependence of the variables $\mathbb{X}$, which is consequence of the manufacturing process. The statistical distribution of $\mathbb{Y}$ is derived from the distribution of $\mathbb{X}$ and can be used to compute the proportion of parts out of specification limits; that is, the non-conforming proportion of parts p. The tolerances of $\mathbb{X}$ are assigned so that they are as wide as possible but keeping the non-conforming proportion p in minimal acceptable values. This statistical criterion is more efficient than using a deterministic criterion. Under a deterministic criterion, also known as worst case tolerancing, the tolerances are assigned so that the 100% of the parts meet all the specifications. This practice leads to tighter tolerances, and hence higher manufacturing costs.

In the field of statistical tolerance synthesis, there is a large number of research works for the case of independence of $\mathbb{X}$. For example, the methodologies proposed in Lee and Woo [1], Lee and Johnson [2], Wu et al. [3], Feng and Balasu [4], Lee and Tang [5], Ye and Salustri [6] and Huang et al. [7]. Although in some cases this assumption could be valid, there are many other cases where variables $\mathbb{X}$ are related. For example, in processes like molding or press stamping the length, width and thickness of a part are all related. In spite of the key importance of the dependence of $\mathbb{X}$ in the tolerance design, the literature based on the dependence of $\mathbb{X}$ is rather scarce.

In Chen [8], a set of independent variables $\mathbb{X}$ become dependent as a result of a selective assembly. For example, a device could be assembled in such a way that parts with high values of X_1 are assembled with parts that have a low value of X_2; that is to say, a negative relationship between X_1 and X_2 is imposed. This selective assembling provokes a correlation that should be taken into account in the optimal tolerance design. Lee et al. [9] propose an optimal tolerance synthesis where the correlation matrix of $\mathbb{X}$ is taken into account. Their non-linear programming algorithm finds the optimal solution by modifying the standard deviations σ_i independently according to so-called feasible directions and keeping the correlation matrix constant. As will be proven in Section 3, this treatment of the dependence structure might not be realistic. In that section, it will be seen that in order to preserve the characteristics of the manufacturing process, the standard deviations σ_i can not be changed independently, since they are linked by the eigenvectors of the covariance matrix.

In this paper, we propose a methodology to allocate optimal statistical tolerances to dependent variables $\mathbb{X}$, where the dependence structure of $\mathbb{X}$ is inherent to the manufacturing process. The manufacturing process is assumed to be defined by a set of independent (latent) factors that explain the dependence structure of $\mathbb{X}$. Under the assumption of normality, these independent factors can easily be computed using principal component analysis (PCA). These independent factors are related to the engineering design of the process and may not be under discussion in the tolerance design. Consequently, the optimal tolerance design should be based on the statistical properties of these independent factors.

The outline of the article is as follows. Section 2 describes briefly the common criteria considered in the current procedures of tolerance synthesis under independence of $\mathbb{X}$. These criteria will be used as a reference point in the proposed methodology. Section 3 describes the basic considerations of the proposed methodology. Section 4 describes the steps of the procedure. Section 5 applies the proposed methodology to two examples cited in the literature, and compares the results with those obtained by assuming independence of $\mathbb{X}$. Section 6 concludes.

2. A review of tolerance synthesis under independence of *X*

2.1. General considerations

Many of the methodologies proposed in the literature under the assumption of independence of $\mathbb{X}$ have some aspects in common. That is the case, for example, of the methodologies proposed in Wu et al. [3], Feng and Balasu [4], Lee and Tang [5], Ye and Salustri [6], Huang et al. [7] and Chen [8]. These common aspects will be described in this section. They will serve as a reference point for the methodology proposed in this article.

Let Y be a characteristic that ensures the functionality of a part. A part is conforming if the variable Y is inside some specifications; that is, $Y \in [LSL, USL]$, where LSL and USL are the lower and the upper specification limits, respectively. These limits are considered symmetric about the mean of Y. It is assumed that under optimal conditions, the non-conforming proportion p is not larger than some small value α; that is

$$p \equiv P(Y \notin [LSL, USL]) \leqslant \alpha \tag{1}$$

The variable Y is a function of $\mathbb{X}$, that represents a set of dimensions of a part or mechanical assembly; that is, $Y = f(X_1, X_2, \ldots, X_K)$. The traditional tolerance synthesis problem consists on finding the optimal tolerances of K independent variables $\mathbb{X} = (X_1, X_2, \ldots, X_K)'$, where the tolerances are of type $X_i \in [LTL_i, UTL_i]$, $i = 1, 2, \ldots, K$. Here, LTL_i is the lower tolerance limit and UTL_i is the upper tolerance limit and both define a symmetric interval about the mean of X_i. These tolerance intervals form a rectangular tolerance region T_X defined as

$$T_X = \left\{ \mathbb{X} \in R^K : (LTL_i \leqslant X_i \leqslant UTL_i),\ i = 1, \ldots, K \right\}. \tag{2}$$

The tolerances are assigned to X_i such that the total cost (manufacturing and quality) is minimized and (1) is attained.