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A continuous model approach for cross-coupled bending vibrations of a rotor-bearing system with a transverse breathing crack

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ABSTRACT

In this paper, the cross-coupled bending vibrations of a rotating shaft, with a breathing crack, mounted in resilient bearings are investigated. The equations of motion of the continuum and isotropic rotating model of the shaft follow the theory of Rayleigh. The governing equations are coupled in the two main directions, and the partial solution is obtained by solving a linear system of equations, for each time step, taking into account the non-linearity due to the breathing crack. The coupling is introduced in three different ways: the equations of motion, the resilient bearings and the crack.

A main focus is made in the coupling introduction due to crack compliance variance while rotation with the cross-coupling terms of the local compliance matrix due to the crack to be calculated analytically as functions of the rotational angle.

The three causes of coupling between the vertical and horizontal vibrations should be distinguished with regard to the effects that each one of them has on the dynamic response of the rotor. Inversely, the existence of each type of coupling in the frequency response could be used to identify the respective cause.

1. Introduction

The vibration of cracked rotors is an issue that has been continuously investigated since 1970. Many turbine rotors failures started appearing in the 1970s in the USA and elsewhere, because many of them were approaching an operating life of 30 years. Such a case was that observed by Dimarogonas in [1,2] and Pafelias [3] at the turbine department of the General Electric Company in Schenectady. That failure was due to a fatigue propagating crack. Since that time many researchers have investigated the dynamic behavior of rotors with open cracks that as phenomenon are close enough to the case of rotors with dissimilar moments of inertia, as Dimentberg [4] and Tondl [5] have extensively treated them. Dimarogonas [6] suggested that the existence of higher harmonics and sub-harmonics, as well as the presence of longitudinal and torsional harmonics in the start-up lateral vibration spectrum due to the coupling, as potential methods for crack detection.

It is known that when a cracked shaft rotates, the stiffness in a fixed direction changes with time due to the local compliance that the crack introduces. The precise computational of local flexibility was confronted by Dimarogonas [1,2] and he gave functions for the change of local compliance during rotation. Chondros and Dimarogonas [7] modeled a transverse crack as a local elasticity and related the crack depth to the decrease in the natural frequency, and Gounaris and Dimarogonas [8] developed a finite cracked Euler–Bernoulli element. The precise model of the crack compliance and the shaft vibration are two objects that have been researched in strong relationship with each other, in order for more precise theories of cracked shaft vibration to be found and a better approximation to the problem of crack identification to be feasible. Jun and Kim [9]

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Nomenclature

R: shaft radius $Y(x,t)$: vertical response
α, α_x: crack depth $Z(x,t)$: horizontal response
$\bar{\alpha}$: dimensionless crack depth $U(x,t)$: complex response
b_i: boundaries of crack in cracked section, $i = 1,2$ $\Theta(x,t)$: slope
$\bar{c}_{ij}$: dimensionless crack compliance $V(x,t)$: shearing force
C_{ij}: crack compliance $M(x,t)$: bending moment
$\bar{\mathbf{C}}$: Local compliance matrix $U_g(x)$: gravity response
C_{tot}: total flexibility $\mathbf{P}$: characteristic matrix
E: Young modulus $\mathbf{K}_b$: bearing stiffness coefficient matrix
G: shear modulus $\mathbf{C}_b$: bearing damping coefficient matrix
g: gravity acceleration t: time
I: shaft polar moment of inertia Δt: time interval
i: complex quantity T: power transmission torque
k: form factor $\Delta\Omega$: rotational speed interval
P_i: bending load v: Poisson ratio
L: shaft length ρ: material density
m_d: disk mass φ: rotational angle
L_d: disk width φ_{cl}: rotational angle of crack closure
R_d: disk radius ω: whirling frequency
r_0: radius of gyration, $\Omega, \overline{\Omega}$: rot, frequency/dimensionless

made a precise study of the free bending vibration of a multi-step rotor using a Timoshenko beam model. Collins et al. [10] used axial impulses in order to detect a breathing crack in a rotating Timoshenko shaft. Collins et al. [11] investigated longitudinal vibrations of a cantilever bar with transverse breathing crack and compared results with those of case of open crack and those without crack. Jun et al. [12] derived the equations of motion of a simple shaft with a breathing crack concluding that the vibration characteristics of a cracked rotor are best identified from the second horizontal harmonic components measured near to the second harmonic resonant speed.

Imam et al. [13], Wauer [14], Gasch [15], Dimarogonas [6], Edwards et al. [16], and Sabnavis et al. [17] presented excellent reviews in the field of dynamics of cracked rotors and suggested different procedures for diagnosing fracture damage. Mayes et al. [18] analyzed the response of a multi-rotor-bearing system containing a transverse crack. When passing through critical speed, the transient vibration of a cracked rotor was analyzed by Sekhar and Prabhu [19], using the finite element method. Ishida and Hirokawa [20] represented the internal resonance of a linear cracked rotor and non-stationary oscillation of a non-linear rotor when passing through the major critical speed. Tsai and Wang [21] presented a free vibration mode analysis of a multi-cracked rotor. Results in many papers were obtained only by computer simulation and experimental studies were relatively few. Lee et al. [22] correlated experimental results, using propagating transverse cracks, with their theoretical analysis. Zhou and Xu [23] demonstrated many non-linear dynamic characteristics of a cracked rotor in their experiment. A recent approach to the problem of cracked rotor dynamics studies the coupling between lateral, axial, and torsional vibrations that the crack provokes. Papadopoulos [24] and Papadopoulos and Dimarogonas [25] studied the coupling of bending and torsional vibrations in a cracked Timoshenko shaft under the assumption that the crack remains open. The presence of bending vibrations in the torsional spectra had been cited as a crack indicator. Papadopoulos and Dimarogonas [26] studied coupling of lateral and longitudinal vibrations and proposed the coexistence of lateral and longitudinal vibration frequencies in the same spectrum as an unambiguous crack indicator. The same phenomenon was confirmed also by Darpe et al. [27] when they analyzed the response of a cracked Jeffcott rotor with a transverse crack in the mid-span. They also calculated [28] the cross-coupling terms of the local compliance matrix as a function of rotational angle of the crack and studied the coupled longitudinal bending and torsional vibrations of a cracked rotor. The case of coupled vibrations of all degrees of freedom has been investigated by Papadopoulos and Dimarogonas [29], and Gounaris and Papadopoulos [30] used coupled response measurements of a rotating Timoshenko shaft to identify a crack.

The aim of the present study is to present a new calculation for the change in the local compliance matrix during rotation. Computation of crack local compliance is done for every shaft rotational step, in order to take into account the exact form of the crack while breathing. Also, the Timoshenko theory for continuous beams is used to model the shaft vibration under the effect of gravity and unbalance. The new model of compliance change approximates the phenomenon of coupling between bending vibrations which exist only in a specific rotational range. The analysis includes the torque that is applied at the ends of the shaft due to power transmission, and the gyroscopic effect due to shaft rotation. Two perpendicular displacements are expressed as one complex displacement and this demands the formulation of complex boundary conditions. The solution of the equation of motion is achieved for every rotational step of the shaft and thus makes the model dependent on time.

For simplicity reasons, the boundary conditions assume rigid bearings at both ends of the shaft and the suitable continuity conditions at the crack position. The coupled linear system of equations with periodically varying boundary conditions becomes solvable for every rotational angle. The characteristic equation can be calculated for every value of rotational speed