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Mechanism analysis of a trisector

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Abstract

This paper presents a graphical procedure for analyzing a trisector – a mechanism used for trisecting an arbitrary acute angle. The trisector employed was a working model designed and built for this purpose. The procedure, when applied to the mechanism at the 60° angle, which has been proven to be not trisectable as well as the 45° angle for benchmarking (since this angle is known to be trisectable), produced results that compared remarkably in both precision and accuracy.

For example, in both cases, the trisection angles found were 20.00000°, and 15.00000°, respectively, as determined by *The Geometer's Sketch Pad software*. Considering the degree of accuracy of these results (i.e. five decimal places) and the fact that it represents the highest level of precision attainable by the software, it is felt that the achievement is noteworthy, notwithstanding the theoretical proofs of Wantzel, Dudley, and others [Underwood Dudley, A Budget of Trisections, Springer-Verlag, New York, 1987; Clarence E. Hall, The equilateral triangle, Engineering Design Graphics Journal 57 (2) (1993); Howard Eves, An Introduction to The History of Mathematics, sixth ed., Saunders College Publishing, Fort Worth, 1990; Henrich Tietze, Famous Problems of Mathematics, Graylock Press, New York, 1965].

Keywords: Mechanism analysis; Angle trisector; Four bar mechanism; Slider crank mechanism; Slider-coupler mechanism; Famous problems in mathematics

1. Introduction

The problem of the trisection of an angle has been for centuries one of the most intriguing geometric challenges for mathematicians [1–4]. According to Underwood Dudley [1] author of *A Budget of Trisections*,

> *"Certain angles can be trisected without difficulty. For example, a right angle can be trisected, since an angle of 30° can be constructed. However, there is no procedure, using only an unmarked straight edge and compasses, to construct one-third of an arbitrary angle".*

Dudley then proceeded to lay out a proof of this statement by showing that a 60° angle cannot be trisected. Also, in the same text, he referenced the work of Pierre Laurent, Wantzel, who in 1837 first proved that such trisection was impossible. Yet in a more recent paper by Hall [2], a proof was presented to show that a three to

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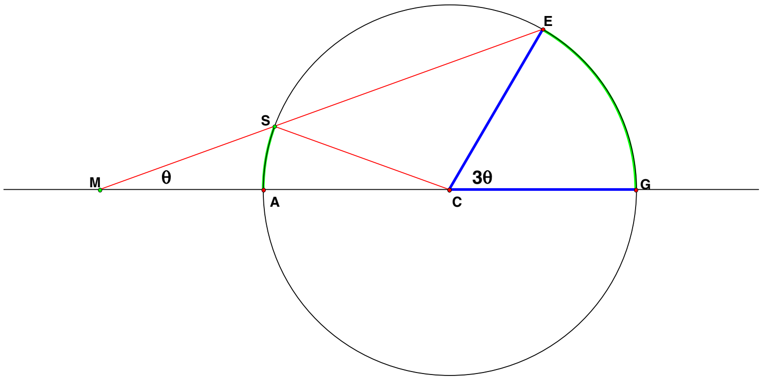
one relationship between certain angles can exist for acute angles. However, Hall [2] did not develop or present a procedure for the trisection.

The purpose of this paper is not to contradict or debate the established proofs alluded to, but to present a summary of the results obtained from my study of a trisector mechanism, which I designed and built as part of the study [5]. It is hoped that these results as well as the approach used in developing them will provide others in the mathematics and science community valuable physical insights into the nature of the problem.

The results presented are based on an analysis, using unmarked ruler and compasses only (aided by *The Geometer's Sketch Pad* software), of the 60° angle that has been proven to be not trisectable as well as the 45° angle that is known to be trisectable.

2. Theory



The proposed method is based on the general theorem relating arcs and angles.

Let ∠ECG (or 3⊖) be the required angle to be trisected. With center at C and radius CE describe a circle. Given that a line from point E can be drawn to cut the circle at S and intersect the extended side GC at some point M such that the distance SM is equal to the radius SC, then from the general theorem relating to arcs and angles,

$$\angle EMG = 1/2(\angle ECG - \angle SCM)$$

$$2\angle EMG + \angle SCM = \angle ECG$$

Since ΔCSM is an isosceles Δ

$$\angle SCM = \angle EMG = \ominus$$

Therefore $3\angle EMG = \angle ECG$ or $3\ominus = \angle ECG$.

3. Trisector design and analysis

The trisector mechanism illustrated in Fig. 1 is a compound mechanism consisting of a four-bar linkage, CEDA, where CE is the crank, ED is the coupler, and DA is the follower, and a slider-crank linkage [6], CFVE″, where CV is the crank, FE″ the connecting rod, and F the slider. The links for the four-bar and slider-crank are designed so that the pin joints are all located at equal distances apart, and both linkages are mounted on a common base and connected at fixed axis C, where the two cranks CE and CV meet, as well as at crank pin E, via a pivoting slot through which the connecting rod slides. Thus, as the four-bar crank CE is rotated in one direction or the other, between the 90° and 0° positions, the connecting rod FE″ of the slider-crank divides the angle formed by said crank and coupler into a 2–1 ratio. Also, if crank CE is rotated, say in a clockwise manner, the connecting rod FE″ would be forced to undergo combined motion of translation, where sliding would occur at both ends, and rotation only at the pivoting slot end. Meanwhile crank CV of the slider-crank would be forced to rotate, but in a counterclockwise manner.

By assuming slider F is not constrained (or not restricted by the slot), while other parts of the mechanism are in motion, the mechanism behaves like a sliding coupler mechanism [6] where, it was possible to show that