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Bayesian uncertainty analysis of finite deformation viscoelasticity

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ABSTRACT

The viscoelasticity of the dielectric elastomer, VHB 4910, is experimentally characterized, modeled, and analyzed using Bayesian uncertainty analysis. Whereas these materials are known for their large-field induced deformation and broad applications in smart structures, the rate-dependent viscoelastic effects are not well understood. To address this issue, we quantify both the hyperelastic and viscoelastic constitutive behavior and use Bayesian uncertainty analysis to assess several key modeling attributes. Specifically, we compare an Ogden-based phenomenological model to a nonaffine hyperelastic model and couple hyperelasticity to both linear and nonlinear viscoelasticity. The utilization of Bayesian statistics is shown to provide insight into quantifying nonlinear viscoelasticity behavior as a function of internal state variables. The results are validated experimentally in the finite deformation regime over a range of stretch rates spanning four orders of magnitude (6.7×10^{-5}–0.67 Hz). A unique set of hyperelastic parameters are identified, independent of the stretch rate. In addition, comparisons of the linear and nonlinear viscoelastic models demonstrate a reduction in modeling error by approximately a factor of three. Finally, the viscoelastic time constant is shown to produce an inverse stretch rate power law dependence regardless of which hyperelastic model is used.

1. Introduction

The utilization of active polymers in adaptive structures is known to provide unique capabilities for real time control of a structure's shape, stiffness, or damping (Smith, 2005). Knowledge of the viscoelastic constitutive behavior over a broad range of deformation rates becomes particularly important in many applications where dynamic tunability or actuator control is of critical importance. Active polymers such as dielectric elastomers, liquid crystal networks, ionic polymers, and nanocomposites often exhibit complicated viscoelastic characteristics that vary over many orders of magnitude of the stretch rate (Rubinstein and Colby, 2003; Drozdov, 1998). The introduction of such a broad range of time scales into a continuum model presents a significant challenge in connecting the underlying material physics with a set of model parameters. Further, in finite deformation regimes, the application of linear or nonlinear viscoelasticity becomes important. The quantification and analysis of model parameter uncertainty is shown to provide important insight towards characterizing this complex material behavior.

Whereas viscoelasticity has been studied extensively (Drozdov, 1998; Bergström and Boyce, 1998; Christensen, 1980; Edwards, 1982; Holzapfel, 1996; Simo, 1987;

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Hilton and Yi, 1998), there still remains significant challenges in accurately quantifying and predicting rate-dependent, finite deformation over a broad range of elastomer deformation rates. The classic formulation utilizes a combination of Maxwell and Kelvin Voigt spring-dashpot models conceptualized from a more general non-conservative thermodynamic framework (Holzapfel, 1996; Holzapfel and Simo, 1996; Gurtin, 1967; Peng et al., 1977). Each discrete linear dashpot is used to quantify rate-dependent stresses as being linear with respect to the stretch rate. Generalizations of this model to three-dimensional thermomechanical deformation with internal order parameters are well summarized by Gurtin (1967) and specific functional forms of the constitutive model have been implemented in finite element codes by Holzapfel and Gasser where they assumed linear viscoelasticity coupled to finite deformation (Holzapfel and Gasser, 2001).

Here we adopt the nonlinear mechanics and thermodynamics approach summarized in Holzapfel and Simo (1996) and Peng et al. (1977) and analyze model and experimental uncertainty when quantifying a set of hyperelastic and viscoelastic parameters. The form of the dissipative energy function and the entropy generation function used in quantifying viscoelasticity are analyzed through comparisons with rate dependent stress-stretch measurements and Bayesian uncertainty analysis. Questions often arise in finite deformation mechanics as to the appropriate model for characterizing the viscous nature of the polymer network during large deformation. It is shown that if the dissipative function is assumed to be proportional to the hyperelastic function (e.g., a form of nonlinear viscoelasticity), the model accuracy improves by a factor of three relative to a linear viscoelastic model for our particular elastomer (VHB 4910 made by 3M).

Classical hyperelasticity is motivated by statistical mechanics which describes a polymer network in terms of changes in configurational entropy (Treloar, 1975). In the undeformed configuration, the polymer is typically assumed to be a Gaussian network (completely random) which results in a neo-Hookean configurational entropy model. Upon large deformation, the polymer chains align reducing the configurational entropy beyond configurations that can be assumed Gaussian. Such behavior is often described by a Langevin function (Treloar, 1975). In most cases, this function only qualitatively matches data. Alternatively, polynomial approximations in terms of principal stretches are used to achieve quantitative predictions of data. Such models include the hyperelastic Ogden, Mooney–Rivlin, or Arruda–Boyce models (Holzapfel, 2000). Critical to the development of these constitutive relations is the assumption of affine deformation in which all cross-linked points in the polymer network displace proportional to the macroscopic stretch. It can be argued that the entanglement and crosslinked network structure of many polymers contain network segments with varying pre-tension that do not deform in an affine manner.

Recent research has focused on using nonaffine deformation to describe reversible hyperelastic material behavior (Rubinstein and Colby, 2003; Davidson and Goulbourne, 2006; Rubinstein and Panyukov, 2002). In particular, Davidson and Goulbourne showed that decomposing hyperelastic stresses into a crosslinked stress and an entanglement stress leads to an accurate prediction of elastomer deformation for both tensile and compressive deformation in different elastomer materials (Davidson and Goulbourne, 2006). Here we first compare this approach to a six parameter Ogden model using Bayesian statistics. Second, we couple hyperelasticity with finite deformation linear and nonlinear viscoelastic models. Bayesian uncertainty analysis is shown to be insightful in assessing assumptions that go into the constitutive model and the choice of linear versus nonlinear viscoelastic models. The robustness of the models is illustrated through quantifying probability distributions of each parameter and how these distributions propagate through the model leading to credible and prediction intervals of stress predictions for a given stretch and stretch rate.

The differences in these models are considered in Section 5 where we explore the coupling between hyperelasticity and viscoelasticity to better understand important rate-dependent material parameters governing hysteresis in elastomers. Specifically, we compare the impact of assumptions associated with affine versus nonaffine polymer chain deformation and extensions to linear and nonlinear viscoelasticity. It is shown that a nonaffine hyperelastic model can give practically the same prediction of finite deformation over a broad range of stretch rates (four orders of magnitude) using half the number of parameters in comparison to a six parameter Ogden hyperelastic model. Further improvements in accuracy are achieved using nonlinear viscoelasticity which assumes dissipation is proportional to the hyperelastic energy function instead of a proportionality to a neo-Hookean free energy density function (linear viscoelasticity).

The numerical algorithm used to sample the set of material parameters, in conjunction with Bayesian statistics, is a Markov Chain Monte Carlo (MCMC) algorithm that uses the Delayed Rejection Adaptive Metropolis (DRAM) method (Haario et al., 2006; Smith, 2014). MCMC involves random sampling of parameter values that yields a distribution for parameters based on a sum-of-squares error between the model and data. The criteria for proposing and accepting a parameter value is based on a distribution rather than simply a decrease in error. From the accepted parameter values, Bayesian statistics allow us to quantify the model parameter distributions instead of quantifying fixed values. Those distributions can subsequently be used to calculate uncertainty propagation in the prediction of stress for a given stretch and stretch rate. Whereas this method has been used in a broad range of disciplines including uncertainty quantification of atomistic potentials (Frederiksen et al., 2004), computational fluid dynamics (Croicu et al., 2012), weather prediction (Wilks, 1995), and engineering structures and design (Bernardini and Tonon, 2010; Mahadevan and Haldar, 2000); less work has been focused on uncertainty analysis of nonlinear continuum based constitutive models (Zhengzheng et al., 2014; Oates, 2014).

In Section 2, we describe the experimental methods and summarize the experimental data. In Section 3, the viscoelastic model is presented including specific hyperelastic